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Financial Market Shocks and the Macroeconomy

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Abstract

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Feedback from stock prices to cash flows occurs because information revealed by firms' stock prices influences the actions of competitors. We explore the implications of feedback within a noisy rational expectations setting where stock prices are affected by fundamental information, observed by some investors, as well as by unobserved shocks to stock market participation. The model is consistent with a number of regularities documented in the macro finance literature and generates new, potentially testable, implications.

1 Introduction

Macroeconomic forecasts, such as the NBER macro forecast, typically use stock returns as a leading economic indicator. This is intuitive. Stock prices are forward looking and should thus reflect expectations of future economic activity. Indeed, a relation between stock returns and expectations of future economic activity is a central feature of the recent literature on consumption-based asset pricing, (e.g., Bansal and Yaron (2004)).[1]

Although the "forward-looking" interpretation of the stock returns/economic activity relation is intuitive, it is not really complete. Stock returns are not claims, for example, on industrial production, which they do predict, but are claims on future dividends. The rationale for the forward-looking view is that returns predict aggregate economic activity because dividends tend to grow when the economy grows. As Cochrane (2011) and others have emphasized,[2] however, the return on the market does not predict future changes in dividends on the market portfolio. If we believe that stock returns predict aggregate economic activity because they are forward looking, then it is somewhat of a puzzle that they do not predict changes in future dividends.

The lack of correlation between stock returns and dividends indicates that a substantial portion of observed stock market volatility is due to changes in discount

[1]See Fama (1990), Schwert (1990), and Choi, Hauser, and Kopecky (1999) for evidence on the relation between stock returns and future economic activity, Barro (1990) for the relation with aggregate investment, and Farmer (2012) for the relation with unemployment. The Conference Board (http://www.conference-board.org/data/bci/index.cfm?id=2160) uses the stock market return as a leading indicator for the overall macroeconomy.

[2]See also Campbell and Shiller (1989) and Cochrane (1996).

rates.[3] Thus, if an increase in stock prices does not reflect an increase in expected dividends then it must necessarily reflect a decline in the discount rate. Given that discount rate changes can have economic consequences, (e.g., through changes in investment choices), it is possible that stock returns are a leading economic indicator because they cause rather than just predict macroeconomic change.

While this story is intuitive, it is also incomplete. Specifically, it does not explain why discount rates (or risk premia) can change without shocks to economic fundamentals. Moreover, in addition to a rationale for exogenous discount rate changes, a complete story requires that we establish a causal relation between stock prices and economic activity, and finally, a reason why the profits and dividends of public corporations are only imperfectly correlated with aggregate economic activity.[4]

To address these issues we develop a model that includes a publicly traded sector as well as private firms, where the latter represent economic agents whose earnings are (endogenously) less than perfectly correlated with the earnings of the public corporations. Because public stock prices convey information, the investment choices of these private entities are influenced by the public firm's stock prices.[5] For tractability

[3]Using a variance decomposition approach, Campbell (1991) finds that cash flow news and discount rate news contribute equally (one-third each) to stock return volatility, with the balance attributable to the covariance between the two types of news.

[4]There is a large class of models where discount rates change as a function of shocks to consumption and expected consumption. See for example, Campbell and Cochrane (1999) and Bansal and Yaron (2004). Researchers, however, have questioned whether these preferences can explain the magnitude of the observed volatility of risk premia (e.g., Hansen, Heaton, and Li (2008), Hansen and Sargent (2007), and Marakani (2009)), and no distinction is drawn between consumption and dividends in these models of changing discount rates.

[5]See Goldstein and Guembel (2008), Ozdenoren and Yuan (2008), and Subrahmanyam and Titman (2001) for other models where there is feedback from stock prices to cash flows via corporate investment. Chen, Goldstein, and Jiang (2007) show that real investment is more sensitive to stock prices when proxies for informed trading are higher, supporting an implication of these models.

reasons we assume that all new investment comes from the private firms rather than the public firms. In addition to generating their own cash flows, these investment expenditures directly affect the cash flows of the public firms. To understand this, consider public firms like IBM and Xerox in the early 1980s facing competition from emerging firms like Microsoft and Apple.

In addition to including two types of firms, the model includes two types of investors, informed and uninformed, and two types of shocks. The first, a technology shock observed by the informed investors, affects the productivity of both the public and private firms. The second, which we describe as a participation shock, exogenously affects the overall demand for traded shares, and thus affects the risk premium in the market. As we discuss below, although our interpretation is somewhat different, we follow Grossman and Stiglitz (1980) and model the participation shock as an exogenous innovation to the supply of public securities that are available to the informed and uninformed investors whose choices we do model.

Because of the trades of the informed investors, stock prices reflect information about the future earnings of the public firms. The information content of the prices, however, is muddled by the shock to market participation that adds noise to the prices. Nevertheless, prices are still a useful input into the investment choices of the private firms. But since the uninformed investors and the non-traded sector cannot discern the extent to which a high price is due to informed trade or a high realization of the participation shock, equilibrium levels of capital investment by the private firms are sensitive to both the technology and participation shocks.

Following Cooper and John (1988) and others, we assume the investment choices of the private firms are strategic complements, which mean that on average the private firms tend to underinvest relative to the social optimum. But, because these firms use the stock prices of the public firms as an indication of future demand, a participation shock that increases the stock prices of public firms increases investment. But, since the private firms compete with the public firms, the participation shock can reduce the profits of public firms. This last component of our model, which implies that there can be negative feedback from the prices of public stocks to their future dividends, dampens the relation between public stock returns and future dividends.

Under the assumption of constant absolute risk aversion (CARA preferences) the model can be solved in closed form and generates, at least qualitatively, the three observations that we mention at the outset. In particular, the model can generate a positive relation between public stock prices and both aggregate economic activity and investment, but a zero or even negative correlation between the public stock prices and the earnings and dividends of the public companies. In addition, our model generates an explicit relation between shocks to stock market participation and future economic activity, illustrating that activity in the financial markets can cause as well as reflect expectations about future aggregate output.

In summary, it is a combination of investment externalities, information asymmetries about technology, and stochastic participation that generate our results. While changes in participation can lead to changes in discount rates and investment choices in settings with symmetric information, the interaction between asymmetrically observed technology shocks and unobserved participation amplifies the participation shocks. Indeed, as we show, shocks to participation have a larger effect on the real economy when the volatility of the technology shocks is larger. We also show that overconfidence of informed agents increases their trading aggressiveness and magnifies the impact of negative feedback on asset prices. Finally, we demonstrate that beliefs about the magnitude of participation shocks can influence their effect on the macroeconomy. Specifically, stronger priors about market efficiency (i.e., that price changes are generated primarily from cash flow news rather than participation shocks) results in greater feedback from participation shocks to the macroeconomy.

Although our model is motivated by a puzzle from the macro finance literature, it draws on a number of different literatures. In particular, we use insights from the literature on economic growth, starting with Schumpeter (1911),[6] and the role of "creative destruction" of existing technologies through innovation. The idea that innovations from emerging firms can push out existing technologies has been around since Schumpeter, but we believe that we are the first to consider how this activity is influenced by activities in the public capital markets.

In addition, we draw on the noisy rational expectations literature started by Grossman (1976) and Grossman and Stiglitz (1980), who were the first to describe the inference problem that arises when there are shocks to supply as well as shocks to expected cash flows. In recent years, this literature and its extensions have been applied to issues that relate to market microstructure, i.e., studies of stock market liquidity and insider trading (viz. Holden and Subrahmanyam, 1996, and Leland, 1992). In this

[6]See, for example, Young (1993) or Durlauf (1991) for discussions of how strategic complementarities can stimulate economic growth.

literature, supply shocks as well as information tend to be interpreted as being very short in duration and affecting individual securities. In contrast, our model is based on the idea that there can be systematic and longer term changes to the demand and supply of stocks that may take a while before their effects are readily apparent to market participants. These changes can arise from innovations at brokerage companies and mutual funds that influence the participation of individual investors as well as from policies that influence the choices of public and private pension funds. While these innovations and policy changes may be observable, their implications for the asset allocation choices of investors may not be apparent until much later.

The plan of our paper is as follows. Section 2 presents the basic model of the traded asset market. Section 3 presents the analysis for the non-traded sector.[7] Section 4 contrasts the case of the social optimum in the real sector with the Nash outcome. Section 5 presents the case where the risk aversion of the uninformed agents differs from that of the informed agents. Section 6 discusses ways in which the effects of participation shocks can be amplified, while Section 7 concludes. All proofs, unless otherwise stated, appear in the Appendix.

2 Asset Market

Our model includes two private firms, one publicly traded firm and a riskless and unlimited storage technology. The public firm consists of an asset in place but no

[7]In this paper, the terms "traded" and "non-traded" refer to whether claims on cash flows of the relevant entity are or are not traded in a financial market, and does not refer to the goods produced by the entity.

growth opportunities.[8] In contrast, the private firms have no assets, but possess a technology that provides the option to invest. The cash flows of the firms are influenced by their own as well as other firms' investment choices. In particular, investments of the private firms can complement each other, and because the private firms may compete with the public firm, higher investment by the private firms can reduce the public firm's final cash flows. We will be referring to the endogenous effect of the private firm's investment choice on the public firm's cash flow as the "feedback" effect.

The public firm is born at date 0, investors trade the stock at date 1, and its cash flows, which are realized at date 2, are expressed as follows,

$$F = \theta + \epsilon + \delta. \tag{1}$$

The variables θ and ϵ represent exogenous technology shocks; ϵ is not revealed until date 2, but θ can be observed by informed investors at date 1. These variables have zero mean and are mutually independent and normally distributed. The endogenous variable δ reflects the investment choices of the private firms. We will temporarily treat δ as exogenously determined by the equation, $\delta = kE(\theta|P)$, but will later confirm that this conjectured relationship holds in equilibrium. If we let $\mu \equiv E(\theta|P)$, so that $\delta = k\mu$ we have

$$F = \theta + \epsilon + k\mu. \tag{2}$$

[8]The assumption that the public firm does not invest was made for tractability reasons. The public firm's investment can potentially dampen the negative feedback from its stock price to its cash flows if the investment is a strategic substitute for the private firm investment. Intuitively, the negative effects of competition from the emerging private firms are reduced if the private firms' investments are crowded out by investment in the publicly traded sector.

Following Grossman and Stiglitz (1980) we assume there is a mass m of informed agents and $1-m$ of uninformed agents, each with negative exponential utility with risk aversion R. Informed agents learn the realization of the technology shock θ perfectly after date 0 and prior to trade at date 1. We also assume that there is an exogenous shock which influences participation in the financial market, and, in turn, affects the supply of shares available to the informed and uninformed investors that we model. As we discussed in the introduction, these shocks, for example, may represent innovations in the brokerage business and policy changes (e.g., constraints on short sales) that have an exogenous effect on supply that cannot be observed. We represent this additional per capita demand by z (or supply by $-z$), which is normally distributed with mean zero, and independent of all other random variables.[9] Throughout the paper we denote the variance of any generic random variable, η, by v_η.

2.1 Equilibrium

Let the subscripts I and U denote the informed and uninformed, respectively. Further, let W_i and ϕ_i, $i = \{I, U\}$, respectively denote the wealth and information sets of the two classes of agents. Each agent solves

$$\max E[-\exp(-RW_i)|\phi_i].$$

Since (as we will show) W_i is normally distributed in equilibrium and so are the information sets ϕ_i, each agent maximizes

$$\max E(W_i|\phi_i) - 0.5\, R \,\mathrm{var}(W_i|\phi_i). \tag{3}$$

[9]The analysis is unchanged if we model z as an shock to the informed agents' endowment.

Let x_i denote the demand of agent i and P the market price. Then $W_i = (F - P)x_i$. It follows from solving (3) that the demand of each informed agent is

$$x_I = \frac{E(F|\theta, P) - P}{R\,\mathrm{var}(F|\theta, P)} = \frac{\theta + k\mu - P}{Rv_\epsilon},$$

and that of each uninformed agent is

$$x_U = \frac{E(F|P) - P}{R\,\mathrm{var}(F|P)} = \frac{(1+k)\mu - P}{R\,\mathrm{var}(F|P)}.$$

Denote $v \equiv \mathrm{var}(F|P) = v_\epsilon + \mathrm{var}(\theta|P)$. The market clearing condition is

$$mx_I + (1-m)x_U + z = 0,$$

or

$$m\frac{\theta + k\mu - P}{Rv_\epsilon} + (1-m)\frac{(1+k)\mu - P}{Rv} + z = 0. \tag{4}$$

Let

$$\mu \equiv a_1\theta + a_2 z. \tag{5}$$

The rational expectations equilibrium of the model is derived in the Appendix, which provides analytic expressions for a_1, a_2, and v, and proves the following proposition.

Proposition 1 *The closed-form expression for the price P is given by*

$$P = H_1\theta + H_2 z, \tag{6}$$

where

$$H_1 = \frac{A}{B}, \tag{7}$$

with

$$\begin{aligned} A \;\equiv\; & m[kmv_\theta(m^2v_\theta + mR^2v_\epsilon v_\theta v_z + R^2v_\epsilon^2 v_z) + \\ & \{mv_\theta + R^2v_\epsilon v_z(v_\epsilon + v_\theta)\}\{m^2v_\theta + R^2v_\epsilon^2 v_z\}], \end{aligned}$$

$$B \equiv (m^2 v_\theta + mR^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z)(m^2 v_\theta + R^2 v_\epsilon^2 v_z),$$

and

$$H_2 = \frac{R v_\epsilon H_1}{m}. \tag{8}$$

Note that H_1, the coefficient of the information variable θ in the equilibrium price, can be positive or negative in equilibrium. Specifically, H_1 is positive if and only if

$$k > -\frac{[m v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta)](m^2 v_\theta + R^2 v_\epsilon^2 v_z)}{m v_\theta (m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z)}. \tag{9}$$

The ambiguity in the sign of H_1 leads to an interesting feature of our model, that the price P can be negatively correlated with θ, the variable representing private information. This "perverse" case, which can arise when high θ is bad news for the public firm, happens when $H_1 < 0$, which implies that the negative effect of the higher investment and increased competition from the private firms more than offsets the direct effect of the higher θ on the public firm's output.[10] While it is interesting that this can happen, within the context of our model this "perverse" case, where good news for the economy is bad news for public companies, does not generate any of our main results.

We now examine price volatility in our setting. From Proposition 1, the variance of the price P can be written as

$$\mathrm{var}(P) = H_1^2 v_\theta + H_2^2 v_z = H_1^2 \left[v_\theta + m^{-2} R^2 v_\epsilon^2 v_z \right]. \tag{10}$$

[10] Note from (1), (2), and the expression for μ in the Appendix, (36), that the feedback component δ is negatively correlated with θ if k is negative. Thus, with sufficiently negative feedback, a high price can be associated with low future cash flows, which leads to a negative correlation between θ and P.

In turn, we have

$$\frac{d\left[\text{var}(P)\right]}{dk} = 2H_1\left[v_\theta + m^{-2}R^2v_\epsilon^2v_z\right]\frac{dH_1}{dk}.$$

Since $dH_1/dk > 0$ (from (7)), the volatility of the public firm's stock price is increasing in the feedback parameter if and only if H_1 is positive (i.e., (9) is satisfied). In other words, when we rule out the perverse case, stronger negative feedback dampens volatility. In the perverse case, however, negative feedback increases volatility.[11]

2.2 Intertemporal Dependence between Cash Flows and Prices

Note that the date 0 price is not stochastic since the information and participation shocks are realized only at date 1. The serial covariance of price changes is therefore $\text{cov}(F-P, P)$. Straightforward calculations lead to the following proposition:

Proposition 2 *For a given level of feedback k, the serial covariance of price changes is given by*

$$-\frac{\nu_1\nu_2}{(m^2v_\theta + mR^2v_\epsilon v_\theta v_z + R^2v_\epsilon^2v_z)^2(m^2v_\theta + R^2v_\epsilon^2v_z)}, \tag{11}$$

where

$$\nu_1 \equiv R^2v_\epsilon^2v_z[kmv_\theta(m^2v_\theta + mR^2v_\epsilon v_\theta v_z + R^2v_\epsilon^2v_z)+$$

$$\{mv_\theta + R^2v_\epsilon v_z(v_\epsilon + v_\theta)\}\{m^2v_\theta + R^2v_\epsilon^2v_z\}],$$

and

$$\nu_2 \equiv m^2v_\theta + R^2v_\epsilon v_z(v_\epsilon + v_\theta).$$

[11]In addition, when there is positive feedback ($k > 0$), volatility increases with greater levels of feedback.

It should be noted that when k is zero (i.e., when there is no feedback from prices to cash flows), the serial correlation is negative. This is a standard result; an unanticipated increase in participation reduces the risk premium, thereby increasing the date 1 price and reducing the expected date 2 return. When there is sufficiently negative feedback (i.e., ν_1 in (11) is negative), however, the serial covariance can be positive. This can arise because the shock to the risk premium has an offsetting effect on the public firm's cash flows. In particular, a participation shock that reduces the risk premium can reduce cash flows sufficiently to cause the public firm's stock price to drop. When this is the case, we will observe low expected returns at date 2 following low realized returns at date 1.

We now turn to the correlation between the realized cash flow of the public firm and the market price. Noting that $\mu \equiv a_1\theta + a_2 z$, and using expressions for a_1 and a_2 in (39) and (40), respectively, within the Appendix, it is easily shown that the covariance $\mathrm{cov}(F, P)$ can be written as:

$$\mathrm{cov}(F, P) = \mathrm{cov}(\theta+\epsilon+k\mu, H_1\theta+H_2 z) = a_1(1+k)H_1 v_\theta + k a_2 H_2 v_z = H_1(1+k)v_\theta. \quad (12)$$

The above covariance can be negative only if H_1 and $1+k$ are of opposing signs. Since the right-hand side of (9) (the bound on k below which H_1 is negative) is less than -1,[12] $1+k$ and H_1 cannot be simultaneously positive and negative, respectively. Thus, we have the following proposition.

Proposition 3 *The closed-form expression for the covariance between the public*

[12] The difference between the absolute value of the numerator and the denominator on the right-hand side of (9) is $m^2R^2v_\epsilon^2 v_\theta v_z + R^4 v_\epsilon^3 v_z^2(v_\epsilon + v_\theta)$, a positive number.

firm's cash flows and the equilibrium price, $\mathrm{cov}(F, P)$*, is given by:*

$$\frac{mv_\theta(k+1)[kmv_\theta(m^2v_\theta + mR^2v_\epsilon v_\theta v_z) + \{mv_\theta + R^2v_\epsilon v_z(v_\epsilon + v_\theta)\}\{m^2v_\theta + R^2v_\epsilon^2v_z\}]}{(m^2v_\theta + mR^2v_\epsilon v_\theta v_z + R^2v_\epsilon^2v_z)(m^2v_\theta + R^2v_\epsilon^2v_z)}.$$

The above covariance is negative if and only if $k < -1$*, and* H_1 *is positive, i.e., (9) holds.*

The idea that the cash flow and price of the publicly traded firm can be negatively correlated is somewhat counterintuitive and arises from a combination of the participation shocks and the feedback. To understand this, note that a positive participation shock will simultaneously increase the public stock price and reduce the public firm's expected cash flow because of the feedback effect. What this means is that if the variance of the participation shock, z, is large relative to the cash flow variances, the net effect is a negative correlation between price changes and future cash flows.[13]

Note that the covariance between the final cash flow and the participation shock z is given by

$$\mathrm{cov}(F, z) = \frac{kRv_\epsilon mv_\theta v_z}{m^2v_\theta + R^2v_\epsilon^2v_z},$$

so that the sign of the covariance depends solely on the sign of k. Thus, with negative feedback, a positive participation shock tends to lower the cash flows generated by the public firm.

[13]To see this, note that in the right-hand side of (9), the highest exponent of v_z is 2 in the numerator, and is unity in the denominator. So as v_z becomes larger, the tendency for (9) to hold becomes stronger.

3 The Non-Traded Sector

Up to this point we have assumed that the returns of the public company have an effect on its cash flows. In this section, we extend the model so that δ, which represents the feedback from the public firm's stock price to its cash flows, is determined endogenously. Specifically, we consider the investment choices of the two private firms, 1 and 2, which produce goods that complement each other's, but which compete with the traded firm's product.[14] We assume that the private firms are both endowed with technologies as well as with a sufficient quantity of an asset that can either be transformed into the investment asset or stored to generate a risk-free cash flow at date 2 (the rate of return on the storage technology is normalized to zero without loss of generality). Each risk-neutral private firm i ($i = 1, 2$) invests capital in the amount of K_i in a non-traded growth opportunity, which produces a product that is either a substitute or a complement to the product of the public firm.

We propose that the profit of Firm 1 depends on the technology shock θ and the level of investment through the function

$$\pi_1 = C_1 + C_2\theta + C_3 K_1 \theta - 0.5(K_1^2 - 2C_4 K_1 K_2). \tag{13}$$

Similarly, the profit of firm 2 is given by

$$\pi_2 = D_1 + D_2\theta + D_3 K_2 \theta - 0.5(K_2^2 - 2D_4 K_1 K_2). \tag{14}$$

The term involving the product of K_1 and K_2 in each profit function captures the strategic complementarity between the two firms. We assume that all of the constants

[14] Extending the model to investment by public firms and many private firms is substantially less tractable, but does not materially alter our substantive insights.

are positive (i.e., $C_i > 0,\ D_i > 0\ \forall i = 1, \ldots, 4$), and $C_4 + D_4 < 1$. The expressions (13) and (14) thus model the notion that profits depend positively on the technology shock θ, and that the marginal productivity of capital is positively related to θ.[15] As we will see, the above formulation implies that in equilibrium, the capital investments of the non-traded sector are linearly increasing in μ, the conditional mean of θ, which is a component of the traded firm's cash flow.[16]

3.1 Equilibrium Investment

This subsection characterizes capital investments of the two private firms, which are determined within the context of a Nash equilibrium where firm i takes the amount invested by firm j as given. Specifically, the private firms maximize the expected value of π_i (conditional on the publicly traded firm's price P), taking the other firm's investment choice as given. Therefore, firm 1 solves

$$\max_{K_1} C_1 + C_2\mu + C_3K_1\mu - 0.5(K_1^2 - 2C_4K_1K_2),$$

while firm 2 solves

$$\max_{K_2} D_1 + D_2\mu + D_3K_2\mu - 0.5(K_2^2 - 2D_4K_1K_2).$$

Performing the maximizations indicated above yields

$$K_1 = \mu C_3 + C_4K_2 \tag{15}$$

[15] We specifically interpret the third component of profits in (13) and (14) as an improvement in product quality and endogenize this term in Section 3.2.

[16] The formulation of the profit function captures the notion that the firms' investments increase in the conditional mean of the traded sector's cash flow component θ and preserve tractability. Alternative specifications are possible, but in most, tractability is lost, hampering closed-form solutions. Subrahmanyam and Titman (1999) use a similar specification.

and

$$K_2 = \mu D_3 + D_4 K_1. \tag{16}$$

Substituting for K_2 from (16) into (15), we have

$$K_1 = \frac{\mu(C_3 + C_4 D_3)}{1 - C_4 D_4}. \tag{17}$$

Similarly,

$$K_2 = \frac{\mu(D_3 + D_4 C_3)}{1 - C_4 D_4}. \tag{18}$$

Thus, the amount of capital invested by each firm is positively correlated with μ, the conditional mean of the traded firm's cash flow component θ. Now, from (17), (18), and the expression for μ in the Appendix, (36), we immediately have the following proposition, stated without proof:

Proposition 4 *In equilibrium, the capital investments of the two private firms are positively correlated both with the participation shock z, as well as the fundamental shock θ.*

Since μ is positively correlated with θ and z, so are K_1 and K_2. Thus, investment varies with shocks to uninformed participation. The sensitivity of the investment to this shock is represented by a_2 in (5), which, from (40) in the Appendix, is increasing in v_θ (the volatility of the technology shock) and decreasing in v_z (the volatility of the participation shock). A high volatility of θ implies that the information content in μ is high, thus raising the sensitivity of investment to μ, and, in turn to z (since μ is linear in z). Also note from (17) and (18) that K_i, $i = 1, 2$ are increasing in C_4 and D_4, the coefficients of the complementary component of profits in (13) and (14);

this captures the intuition that strategic complementarities increase investment by the private firms.

Substituting for K_1 and K_2 from (17) and (18) into π_1 and π_2, we have that

$$\pi_1 = C_1+\theta C_2+\frac{(C_3 + C_4D_3)[2\mu\theta C_3(1 - C_4D_4) + \mu^2\{C_4D_3 + C_3(2C_4D_4 - 1)\}]}{2(1 - C_4D_4)^2}, \quad (19)$$

and

$$\pi_2 = D_1 + \theta D_2 + \frac{(D_3 + D_4C_3)[2\mu\theta D_3(1 - C_4D_4) + \mu^2\{D_4C_3 + D_3(2C_4D_4 - 1\}]}{2(1 - C_4D_4)^2}. \quad (20)$$

The realized profits vary both with θ and μ, the latter because the capital invested is a linear function of μ. It can easily be shown that the expected profits conditional on the public firm's stock price P are positive and increasing in the constants C_3, D_3, C_4, and D_4, where the former two and the latter two parameters measure the marginal productivity of capital and the degree of strategic complementarity, respectively.

3.2 Feedback to the Traded Sector

We wish to capture the notion that there are emerging firms in the private sector with products that can either complement or substitute for the products produced by established firms in the public sector. For example, a firm's investment may improve the quality of a product that complements the public firm's product or alternatively, it may increase the quality of a competing product. We model this by postulating that firm 1's product is complementary to that of the traded firm whereas firm 2's product is a differentiated substitute.

Specifically, suppose that χ_i, ζ_i, $i = 1, 2$ are positive constants. We assume that each unit of capital invested by firm 1 increases the quality of firm 1's product by $\chi_1 + C_3\theta$, so that an investment K_1 implies a quality improvement of $K_1\chi_1 + C_3K_1\theta$. While the second component increases firm 1's profit one-to-one (as in the third term of (13)), the first component increases the traded firm's profit by ζ_1 units. Let $G_1 \equiv \chi_1\zeta_1$. Then, each unit of capital invested by firm 1 increases the traded firm's profit by G_1 units. Similarly, an investment of K_2 generates a quality improvement of $K_2\chi_2 + D_3K_2\theta$, where each unit of the first component decreases firm 1's profit by ζ_2 units. Letting $G_2 \equiv \chi_2\zeta_2$, the feedback component of cash flow δ in (1) is given by

$$\delta = G_1K_1 - G_2K_2, \tag{21}$$

where $G_1 > 0$ and $G_2 > 0$.

From (17) and (18), the above formulation implies that

$$k = \frac{G_1(C_3 + C_4D_3) - G_2(D_3 + D_4C_3]}{1 - C_4D_4}. \tag{22}$$

We then have the following proposition.

Proposition 5 *The sign of the correlation between the cash flows of the traded firm and the participation shock z, $\mathrm{corr}(F, z)$, is ambiguous and is negative if and only if*

$$G_1(C_3 + C_4D_3) < G_2(D_3 + D_4C_3). \tag{23}$$

Essentially, a positive participation shock tends to raise the level of investment by both firms. Whether the participation shock positively or negatively influences the public firm's cash flows depends on whether the complementary or competitive influences of the non-traded firms dominate. Note that the greater is the effect of competition from firm 2 (represented by the parameter G_2), the greater is the tendency for $\mathrm{corr}(F, z)$ to be negative. Also, rewriting (23) as

$$C_3(G_1 - G_2C_4) < D_3(G_2 - G_1C_4),$$

we see that for sufficiently large G_2, greater productivity of the competing firm's capital (represented by D_3) increases the tendency for (23) to hold. Note that if the non-traded firms *both* compete with the traded firm (i.e., if $G_1 < 0$), then a participation shock always has a negative influence on the traded firm's cash flow.

We now analyze the correlation between the stock price of the public firm and total cash flows in the economy. Let

$$q \equiv C_2 + D_2, \tag{24}$$

and let $T \equiv F + \pi_1 + \pi_2$ denote the total cash flows in the economy. Then, from (2), (19), and (20), we have that

$$\begin{aligned} \mathrm{cov}(T, P) &= \mathrm{cov}[\theta(1+q) + \epsilon + k\mu, H_1\theta + H_2 z] = (1 + q + ka_1)H_1 v_\theta + ka_2 H_2 v_z \\ &= H_1(1 + k + q)v_\theta. \end{aligned} \tag{25}$$

This immediately leads to the following proposition, stated without proof.

Proposition 6 *Provided that $H_1 > 0$, the correlation between public stock prices and the economy's total cash flows is positive if and only if*

$$k > -(1+q),$$

i.e., if and only if the right-hand side of (22) is greater than $-(1+q)$.

Since G_1 and G_2 are exogenous, and the right-hand side variables in (9), the condition which determines the sign of H_1, do not involve G_1 and G_2, it is always possible to choose quantities such that the conditions in the above proposition are satisfied. As the proposition demonstrates, feedback has to be sufficiently negative for total cash flows to move inversely with market prices. The bound on k in Proposition 6 is less strict than the bound $k > -1$ in Proposition 3. This is because, compared to the last expression in (12) for $\text{cov}(F, P)$, there is an extra positive term $q\, v_\theta$ in the expression for $\text{cov}(T, P)$ within (25).

As a numerical example, consider the parameter values $C_1 = D_1 = 0$, $C_2 = D_2 = C_3 = 1$, and $D_3 = C_4 = D_4 = 0.4$. Also suppose that the exogenous variances (v_θ, v_ϵ, and v_z) and the risk aversion coefficient R all equal unity and the mass of informed agents $m = 0.5$. Since we use this parameter set in other parts of the paper (and vary k through the exogenous parameters G_1 and G_2), we will conveniently denote the set as Ω.

To illustrate the different signs and magnitudes of $\text{corr}(F, P)$ and $\text{corr}(T, P)$ as a function of the level of feedback, we use the parameter set Ω, and let $G_1 = 1$, while varying G_2 from 2 to 3, implying a variation in k from -0.52 to -1.48. Figure 1 demonstrates how the correlation between traded cash flows and prices, $\text{corr}(F, P)$ switches from positive to negative as k decreases, but the correlation between total cash flows and prices, $\text{corr}(T, P)$ is positive throughout this range of k.

The following proposition on the correlation between prices, cash flows, and investment can also be derived from the preceding analysis.

Proposition 7 *Suppose that $H_1 > 0$ (i.e., (9) is satisfied), and G_1 and G_2 are such that $-1 > k > -(1+q)$, i.e., the right-hand side of (22) falls between -1 and $-(1+q)$. Then, in equilibrium, (i) the correlation between the traded firm's cash flow and its stock price is negative, i.e.,* $\mathrm{corr}(F, P) < 0$*, (ii) the correlation between aggregate investment and the public stock price, and aggregate cash flow and the public stock price are both positive, i.e.,* $\mathrm{corr}(K_1 + K_2, P) > 0$ *and* $\mathrm{corr}(T, P) > 0$.

Thus, our model is simultaneously consistent with a low or negative correlation between future dividends and current stock prices, and a positive correlation between future economic output and current stock prices. The economic intuition behind Proposition 7 is as follows. Higher prices stimulate greater investment by the non-traded sector, generating competition from the non-traded firms that can lead to a negative relation between the traded sector's cash flows and its stock prices. In the overall economy, the effect of greater investment in the non-traded sector dominates the effect of increased competition in the publicly-traded sector, so that the combined (aggregate) cash flow of the traded and the non-traded sector (i.e., the aggregate output) is positively correlated with prices.

3.3 Implications

This section describes some potentially testable implications of our analysis. Broadly speaking, an interesting area of future research is on how the informational efficiency of the stock market influences the relation between stock market returns and future economic output and capital investment expenditures. Specifically, we consider the effect of more volatile participation shocks, which make stock prices less informative, and a higher concentration of informed investors, which makes prices more informative. The following proposition, which considers regressions of aggregate output and investment expenditures on stock prices, is proved in the appendix.

Proposition 8 *Let* $\beta_T \equiv \mathrm{cov}(T, P)/\mathrm{var}(P)$ *and* $\beta_K \equiv \mathrm{cov}(K_1 + K_2, P)/\mathrm{var}(P)$ *respectively represent the sensitivities of aggregate output and real investment expenditures to stock market prices. Then, under the conditions of Proposition 7, for* $i = \{T, K\}$*, we have that* $\beta_i > 0$*,* $d\beta_i/dv_z < 0$*,* $d\beta_i/dv_\epsilon < 0$*, and* $d\beta_i/dm > 0$*.*

As the above proposition demonstrates, the sensitivities of aggregate output and real investment expenditures to stock market prices are positive and declining in the variance of participation shocks v_z and the risk borne by the informed v_ϵ, and increasing in the mass of informed investors m. The intuition is that prices reflect more information when informed trading is more intense (higher m or lower v_ϵ) and the noise in market prices (v_z) is not too high and this increases the sensitivity of investment as well as output to prices.

Also, in our model the correlation between stock prices and dividends is weakened by negative feedback from the privately held firms, so a large privately held sector is a necessary condition for a weak correlation between stock prices and dividends.

The above observations can potentially be tested by looking at the relation between stock returns and investment expenditures and aggregate output across countries. Specifically, the preceding discussion suggests the following:

1. Aggregate output and capital investment are likely to be less sensitive to stock prices in economies with a larger mass of unsophisticated (e.g., retail) investor participation and a smaller mass of informed investors (e.g., institutional).

2. In general, economies with institutions that facilitate the dissemination of information will exhibit a stronger relation between stock prices and aggregate output. For example, the relation between total economic output and stock market prices will be greater for economies with an active community of equity analysts.

3. Economies with greater inherent uncertainty (due, for example, to political risk) will have weaker links between total output and stock price levels.

4. The relation between stock prices and future dividends is likely to be weaker in countries with an emerging private sector with investment expenditures that are either directly or indirectly funded by the stock market. For example, the association between stock returns and dividends may be weaker in countries with an active venture capital industry and an active IPO market.[17]

[17]See Michelacci and Suarez (2004) for a model that links the funding of emerging firms by venture

4 Social Optimum

This section compares the social optimum to the Nash equilibrium described in the previous section. We do this by first examining whether the level of investment in the Nash equilibrium is above or below the social optimum in the case with complete information. As we show, when there is positive feedback, the Nash equilibrium is characterized by underinvestment. This follows because the investment choices of the two private firms are complements, and the complementarity of one of the private firms with the public firm's output outweighs the effect of the other firms substitute output. In the case with negative feedback the effect can go either way. Again, the complementarity of the private firms creates a tendency for underinvestment, but the negative feedback creates a tendency for overinvestment, since the investment expenditures of the private firms has a negative effect on the public firm.

We then compare the Nash equilibrium with noisy rational expectations to the full revelation social optimum. We particularly focus on a case where the Nash equilibrium generates underinvestment when the participation shock is exactly zero. When this is the case, a small positive participation shock results in higher investment and increases the economy's cash flows, and is thus welfare improving. In this case, a negative participation shock has the opposite effect and a sufficiently large participation shock can lead to overinvestment relative to the social optimum.

To illustrate these results, we consider a social planner who maximizes the sum

capitalists to activities in the stock market and the market for IPOs.

of the firms' conditional expected profits.[18] Thus, the planner solves

$$\max_{K_1,K_2} E(F+\pi_1+\pi_2|P) = \max_{K_1,K_2} \mu\,[C_3' K_1 + D_3' K_2] - 0.5[K_1^2 + K_2^2 - 2(C_4+D_4)K_1K_2], \tag{26}$$

where we define $C_3' \equiv C_3 + G_1$ and $D_3' \equiv D_3 - G_2$. Setting to zero the partial derivatives of the above expression with respect to K_1 and K_2 yields

$$K_1 \;=\; \mu C_3' + (C_4+D_4)K_2, \tag{27}$$

and

$$K_2 \;=\; \mu D_3' + (C_4+D_4)K_1. \tag{28}$$

Substituting for K_2 from (28) into (27), yields,[19]

$$K_{o1} = \frac{\mu\,[C_3' + (C_4+D_4)D_3']}{1-(C_4+D_4)^2}, \tag{29}$$

and similarly

$$K_{o2} = \frac{\mu\,[D_3' + (C_4+D_4)C_3']}{1-(C_4+D_4)^2}, \tag{30}$$

where the additional subscript o denotes the social optimum.

Comparing (17) to (29) and (18) to (30), it follows that if $G_2 = 0$, so that there is no negative feedback, then $\mu^{-1}[K_{oi} - K_i] > 0 \quad \forall i = 1,2$.[20] That is, without

[18]It can easily be shown that the profits of the informed and uninformed (and therefore, the trading costs of the agents with exogenous demand z) do not depend on the level of feedback and, in turn, on the level of investment. The feedback-dependent part of the wealth is $k\mu - P = (ka_1 - H_1)\theta + (ka_2 - H_2)z$. From (7), (8), as well as (39) and (40) in the Appendix, $ka_1 - H_1$ and $ka_2 - H_2$ do not involve k. Therefore, the expected utilities of the agents who trade the public claim do not form part of the social objective.

[19]It is readily verified that the assumption $C_4 + D_4 < 1$ ensures the negative definiteness of the Hessian matrix, guaranteeing a maximum.

[20]To see this, first note that the denominators in (29) and (30) are smaller than their respective counterparts in (17) and (18). If $G_2 = 0$, then the terms multiplying μ in numerators of (29) and (30) are greater than the corresponding terms in (17) and (18).

negative feedback, there is underinvestment in the Nash equilibrium relative to the social optimum (too little investment in good scenarios with positive μ and too little divestment in bad scenarios with negative μ). But, with negative feedback, for any $\mu > 0$, the social planner may choose investment levels lower than the Nash outcome to mitigate the impact of such feedback on the publicly traded firm's cash flow. In this scenario, there will still be underinvestment as long as the feedback is not too negative and the strategic complementarity between the private firms is sufficiently high.[21]

From (2), (13), and (14), the total cash flows of the economy, denoted by π_o, are then given by

$$\begin{aligned}\pi_o &= C_1 + D_1 + \epsilon + k_o\mu + (1 + C_2 + D_2 + C_3K_{o1} + D_3K_{o2})\theta \\ &- 0.5\left[K_{o1}^2 + K_{o2}^2 - 2(C_4 + D_4)K_{o1}K_{o2}\right],\end{aligned}$$

where $k_o \equiv G_1K_{o1} - G_2K_{o2}$. Note that when θ is publicly revealed prior to investment, $\mu = \theta$. Denote the right hand side of the above expression when $\mu = \theta$ as $\pi_{fo}(\theta)$, where the subscript f denotes full revelation of θ. Further, for a given level of μ and z, denote the final cash flow of the privately held and publicly traded firms at the Nash equilibrium level of capital as $\pi_n(\mu|z)$ and $F_n(\mu|z)$, respectively. We then have the following proposition.

[21]Comparing (17) to (29), the coefficient of μ in the latter is less than that in the former if and only if the inequality $G_2 < \frac{C_3(C_4+D_4)^2+C_4^2D_3(C_4+D_4)+G_1(1-C_4D_4)+D_3D_4}{(C_4+D_4)(1-C_4D_4)}$ is satisfied. As the complementarity parameters C_4 and D_4 approach their upper bound of unity from below, the right-hand side becomes ever larger increasing the tendency for this inequality to hold. An analogous argument holds for the comparison of (18) to (30).

Proposition 9 *Suppose that $\theta > 0$ and*

$$\pi_{fo}(\theta) > \pi_n(\mu|z=0) + F_n(\mu|z=0). \tag{31}$$

Then, relative to a Nash equilibrium with a zero participation shock, an arbitrarily small participation shock $z = z_c > 0$ causes the economy's cash flow to increase towards its socially optimal, full-revelation level.

We now demonstrate via a numerical example how participation shocks can move the overall economy closer to the social optimum with full revelation. Consider the parameter set Ω of the previous section with G_1 and G_2 fixed at 1 and 2, respectively and assume that $\theta = 1$ and $\epsilon = 2$. For this set of parameters, in the full revelation, social optimum case, we have $K_{o1} = 2.1$, $K_{o2} = 1.3$ and $\pi_{fo}(\theta) = 6.39$. In the Nash equilibrium with $z = 0$, however, we have $K_1 = 0.28$, $K_2 = 0.19$, and $\pi_n(\mu|z = 0) + F_n(\mu|z = 0) = 5.33$. In the Nash equilibrium with $z = 0.5$, $K_1 = 0.55$, $K_2 = 0.38$, and $\pi_n(\mu|z = 0.5) + F_n(\mu|z = 0.5) = 5.59$. Thus, a positive participation shock shifts the economy's output closer to its socially optimal level under full revelation.[22]

5 Different Risk Aversion of the Informed and Uninformed

Up to now, the analysis assumes that the uninformed and informed agents are equally risk averse. We now relax this assumption and describe the equilibrium where the

[22] As mentioned at the beginning of this section, a sufficiently large participation shock can reverse this result. For example, if $z = 8$, $\pi_n(\mu) + F_n(\mu)$ becomes 0.79, lower than the social optimum of 6.39. Similarly, a large negative participation shock will also drop output below the full revelation social optimum. Thus, if $z = -4$, $\pi_n(\mu) + F_n(\mu)$ drops to 0.67, also well below the social optimum.

risk aversion coefficients of the informed and uninformed differ. Let the risk aversion coefficient of the uninformed agents be denoted by R_U and let R continue to denote the risk aversion of the informed. The market clearing condition (4) now becomes:

$$m\frac{\theta + k\mu - P}{Rv_\epsilon} + (1-m)\frac{(1+k)\mu - P}{R_U v} + z = 0. \tag{32}$$

The Appendix proves the following proposition.

Proposition 10 *When uninformed agents have a risk aversion coefficient R_U that is different from the risk aversion of the informed, R, the equilibrium price $P = H_1'\theta + H_2'z$, where the coefficient H_1' is given by*

$$H_1' \equiv \frac{A'}{B'}, \tag{33}$$

where

$$\begin{aligned} A' &\equiv m[kmv_\theta\{m^3v_\theta(R-R_U) - m^2Rv_\theta + mR^2v_\epsilon v_z(Rv_\epsilon - R_U(v_\epsilon + v_\theta)) - R^3v_\epsilon^2v_z\} \\ &+ (m^2v_\theta + R^2v_\epsilon^2v_z)\{m^2v_\theta(R-R_U) - mRv_\theta - R^2R_Uv_\epsilon v_z(v_\epsilon + v_\theta)\}] \end{aligned}$$

and

$$B' \equiv (m^2v_\theta + R^2v_\epsilon^2v_z)[m^3v_\theta(R-R_U) - m^2Rv_\theta + mR^2v_\epsilon v_z\{Rv_\epsilon - R_U(v_\epsilon + v_\theta)\} - R^3v_\epsilon^2v_z].$$

The coefficient H_2' equals $Rv_\epsilon H_1'/m$.

Interestingly, the difference between H_1 and H_1' does not depend on k. Indeed, suppose that $R_U = \rho R$. Then, from (7) and (33), we have

$$H_1' - H_1 = \frac{(\rho-1)(1-m)mR^2v_\epsilon^2v_z[m^2v_\theta + R^2v_\epsilon^2v_z(v_\epsilon + v_\theta)]}{B''} \tag{34}$$

where

$$\begin{aligned} B'' &\equiv [m\rho\{m^2 v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta)\} + (1-m)(m^2 v_\theta + R^2 v_\epsilon^2 v_z)] \\ &\times [m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z] > 0. \end{aligned}$$

As can be seen, the right-hand side of (34) does not involve k. Thus, since $H_1 - H_1'$ and $H_2 - H_2'$ are both invariant to k,[23] the difference between equilibrium prices when $R_U = R$ and $R_U \neq R$ does not depend on k for given realizations of θ and z. Both cash flows and price involve k, and k affects these quantities and, in turn, the demands of the agents in a way that the price as a function of R_U (for a given R) is invariant to k. Also note from (34) that if $\rho > 1$, i.e., if $R < R_U$, $H_1' > H_1$. The intuition is that if the informed agents are less risk averse than the uninformed ones, they trade more aggressively on their information and this increases the loading of the price on their information θ.

We now present comparative statics associated with $\mathrm{corr}(F, P)$, the correlation between prices and cash flows of the traded firm. We use the parameter set Ω, for every exogenous parameter except the one being varied for the comparative static, and assume $R_U = 1$, $G_1 = 1$, and $G_2 = 3$. Figures 2 and 3 respectively demonstrate that the correlation becomes progressively less negative as the risk aversion of the informed decreases relative to the uninformed, and as the post-date 1 risk of the informed, v_ϵ, decreases. The reason is that the initial price response to a liquidity shock, which causes the negative correlation via negative feedback, depends on the trading aggressiveness of the informed, which in turn depends inversely on the risk aversion of and the risk borne by the agents. Figure 4 demonstrates that the corre-

[23]Note that $H_2 = R v_\epsilon H_1 / m$ and $H_2' = R v_\epsilon H_1' / m$, so that if $H_1 - H_1'$ is invariant to k, so is $H_2 - H_2'$.

lation also becomes less negative as the variance of the participation shock increases. An increase in this variance (v_z) decreases the signal-to-noise ratio in the market price, which makes the price less responsive to a liquidity shock, thus decreasing the effect of negative feedback on the traded firm's cash flows.

6 Amplification of Participation Shocks

Up to this point we have presented a simple model that can be solved in closed form and generates a number of qualitative results that are consistent with the empirical macro finance literature. In particular, the model can generate zero correlation between stock returns and subsequent dividend changes but a positive correlation between stock prices and aggregate economic activity. In addition, the model generates negative serial correlation in aggregate stock returns (or equivalently the predictability of returns with price scaled ratios). Within the context of our model, a crucial attribute that determines the strengths of these relations is the participation shock, suggesting that the observed magnitude of predictability in the data requires large and uncertain participation shocks.

In this section we consider channels that might amplify the effect of participation shocks. We first consider the possibility that informed investors are overconfident about the precision of their information. As we show, overconfidence amplifies the participation shock, thereby increasing the strength of the relation between aggregate output and market prices. We then briefly discuss parameter uncertainty; specifically about how our results may be altered if the variance of the participation shock is unknown.

6.1 Overconfident Investors

We first consider the possibility that informed investors are overconfident.[24] Specifically, they believe that their information about the technology shock is more precise than it really is, i.e., they underestimate v_ϵ to be $v_c < v_\epsilon$. Overconfidence makes the informed agents more aggressive, which in turn makes the price more sensitive to participation shocks. Thus, an increase in the level of overconfidence amplifies the effect of feedback and increases the magnitude of the correlation between the public firm's cash flows and stock prices. The Appendix proves the following proposition.

Proposition 11 *Consider a scenario where informed agents rationally assess all the model's parameters. As one moves away from this setting to one of increasing overconfidence, i.e., v_c, the estimate of v_ϵ is progressively lowered, the absolute magnitude of* $\text{corr}(F, P)$*, the correlation between the traded cash flows and the stock price, increases.*

6.2 Parameter Uncertainty

As we show in the previous subsection, participation shocks can be amplified if informed investors are overconfident. In this section we describe how uncertainty about the expected magnitudes of participation and technology shocks can also amplify

[24] See Odean (1998) for an excellent review of the extensive literature documenting the pervasive overconfidence bias.

participation shocks. Because the model cannot be solved in closed form without common knowledge about the magnitude of these shocks, we provide intuition, i.e., conjectures, in a stylized setting.

Consider, for example, a setting where the true volatility of the participation shock is drawn from a distribution of possible volatilities. While this distribution is common knowledge, the actual volatility that is drawn is not observed by market participants. We conjecture that in this setting, the magnitude of the negative serial correlation of returns increases with the volatility of the participation shock that is drawn.

Unfortunately, solving a model where agents draw parameter values from a prior distribution is quite challenging because in this case, the linearity of conditional expectations formed by the uninformed would be lost (this linearity holds only when variances and unconditional means are non-stochastic). Our intuition comes from the following proposition.

Proposition 12 *If investors irrationally believe that the variance of the participation shock is lower than its actual value, then the volatility of the equilibrium price and the magnitude of its serial correlation will be greater.*

Intuitively, prices are more volatile when investors underestimate the volatility of the participation shock because they tend to underweight the possibility that price changes reflect changes in risk premia rather than changes in expected cash flows, and hence reduce the extent to which they take actions that offset the participation shocks, i.e., buying the stock when the supply is higher and vice versa. The above proposition should be viewed as "suggestive," however, since it considers an investor prior that is simultaneously a point estimate and is wrong, which is inconsistent with rationality.[25]

7 Conclusion

Motivated initially by the equity premium puzzle described by Mehra and Prescott (1985), researchers have struggled with a number of features of the data that relate financial market prices to the macroeconomy. In addition to the magnitude of the equity premium, researchers have developed models to address the time series properties of default free interest rates and expected equity returns, and how they relate to dividends and aggregate consumption. To a large extent, the goal of this research has been to identify a plausible characterization of preferences that are consistent with the data.

This paper takes an alternative approach, exploring a subset of the issues considered in the macro finance literature with a very different type of model: noisy rational expectations with asymmetric information. In particular, we extend Grossman and Stiglitz (1980) to evaluate how stock price movements caused by uninformed participation shocks can influence the profits of public companies as well as overall economic

[25]Solving an equilibrium where the degree of uncertainty about participation shocks are drawn from a distribution creates considerable challenges within our setting since prices in this setting are no longer normally distributed. But, it is likely that the above intuition can be captured numerically in a simpler setting with specific numerical values. For example, consider the case where the investor prior distribution is that the standard deviation of the participation shock is 1 with a probability of 0.99 and 10 with a probability of 0.01, but where the actual standard deviation of the participation shock is 10. We conjecture that the equilibrium price in this setting is very close to that in the ad hoc equilibrium described in Proposition 12.

activity. Within the context of this model we endogenously generate the positive correlations between stock returns and aggregate economic activity, and stock returns and investment, which are observed in the data, as well as the lack of correlation (or even negative correlation) between stock returns and dividends.

In our model, investment by private firms allows them to better compete with publicly traded firms. Since these investments tend to increase with the stock price of the traded firm, they reduce the cash flows of publicly traded firms, generating a weak or even negative correlation between the public firms' cash flows and their stock prices. In this setting the correlation between aggregate output and stock prices remains positive, because the non-traded sector's cash flows are positively related to the technology shocks, which are also reflected in stock prices. Our results are thus consistent with the documented insignificant relation between cash flows and dividends discussed, for example, in Fama and French (1988), Campbell and Shiller (1989) and Cochrane (2011), as well as a significantly positive relation between aggregate macroeconomic production and stock prices. We develop additional implications that relate proxies for informational efficiency to the strength of the relation between stock prices and total output.

At this point, our research agenda is much less ambitious than the preference-based macro finance literature. In particular, we address fewer facts, and our focus is on qualitative results based on a closed form model with CARA preferences, and we thus do not address issues that relate to economic magnitudes. Nonetheless, our results identify various levers that may help future researchers who are interested in quantitative as well as qualitative results.

In addition to the parameters that we explicitly consider in our model, our analysis suggests potential adaptations of the model that may help future researchers match the observed moments in the data. For example, our analysis suggests that overconfident informed investors react more aggressively to information, increasing the correlation between stock prices and economic output. In addition, the volatility of stock prices is influenced by the beliefs of the uninformed investors about the volatility of participation shocks. Thus, when uninformed investors believe that participation shocks are less volatile than they really are, stock prices will be more volatile. Finally, the volatility and serial correlation of returns are likely to be amplified if investors have preferences that make them more risk averse when they are less wealthy.[26] Whether or not deviations from rational expectations or changes in preferences can be exploited to yield economic moments that match the moments observed in the data is a challenge that warrants future research.

[26]Unfortunately, a closed form solution of our model requires CARA preferences.

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Appendix

Proof of Proposition 1: As part of their optimization, the uninformed agents solve a filtration problem which infers θ from the price P, which is a linear combination of θ and z. Let

$$\tau \equiv \frac{m\theta}{Rv_\epsilon} + z.$$

Note that (4) can be solved for P and written as

$$P = [mv + (1-m)v_\epsilon]^{-1}\left[Rvv_\epsilon\tau + \mu\{kmv + (1-m)(1+k)v_\epsilon\}\right]. \tag{35}$$

Since μ and v are non-stochastic from the uninformed's perspective, P is observationally equivalent to τ. Thus, we have

$$\mu = E(\theta|\tau) = \frac{Rmv_\theta v_\epsilon}{m^2 v_\theta + R^2 v_\epsilon^2 v_z}\tau, \tag{36}$$

and

$$v = v_\epsilon + v_\theta - \frac{m^2 v_\theta^2}{m^2 v_\theta + R^2 v_\epsilon^2 v_z}. \tag{37}$$

Note that μ can be written as

$$\mu = a_1\theta + a_2 z \tag{38}$$

where

$$a_1 = \frac{m^2 v_\theta}{m^2 v_\theta + R^2 v_\epsilon^2 v_z} \tag{39}$$

and

$$a_2 = \frac{Rmv_\theta v_\epsilon}{m^2 v_\theta + R^2 v_\epsilon^2 v_z}. \tag{40}$$

Note that the bigger is v_θ, the bigger are the coefficients a_1 and a_2. Thus a given informational or participation shock has a bigger impact on cash flows if the volatility of the information variable is higher.

Solving for P from (4), we have

$$P = \frac{Rvv_\epsilon z + mv\theta + \mu[k\{m(v - v_\epsilon) + v_\epsilon\} + v_\epsilon(1 - m)]}{m(v - v_\epsilon) + v_\epsilon}.$$

Substituting for μ and v from (36) and (37), respectively, we have the expression in Proposition 6. ‖

Proofs of Propositions 2 and 3: Now, we have that

$$\begin{aligned} \mathrm{cov}(P_2 - P_1, P_1 - P_0) &= \mathrm{cov}(\theta + k\mu - H_1\theta - H_2 z, H_1\theta + H_2 z) \\ &= (1 + ka_1 - H_1)H_1 v_\theta + (ka_2 - H_2)H_2 v_z. \end{aligned} \tag{41}$$

Similarly, we have

$$\mathrm{cov}(F, P) = H_1(1 + ka_1)v_\theta + ka_2 H_2 v_z. \tag{42}$$

Substituting for H_1 and H_2 from (7) and (8), and for a_1 and a_2 from (39) and (40), respectively, into (41) and (42) yields the expressions in (11) and (12), which, in turn, lead to Propositions 2 and 3, respectively. ‖

Proof of Proposition 5: Substituting for K_1 and K_2 from (17) and (18), respectively into (21), we find that the equilibrium value of δ is

$$\delta = \frac{\mu[G_1(C_3 + C_4D_3) - G_2(D_3 + D_4C_3)]}{1 - C_4D_4}. \tag{43}$$

so that

$$k = \frac{\mu[G_1(C_3 + C_4D_3) - G_2(D_3 + D_4C_3)]}{1 - C_4D_4}$$

in (2). Substituting for δ from (43) into (1), and using the expression for μ from (38), the equilibrium cash flows of the firm are given by

$$F = \theta + \epsilon + \frac{G_1(C_3 + C_4D_3) - G_2(D_3 + D_4C_3)}{1 - C_4D_4}(a_1\theta + a_2 z).$$

where a_1 and a_2 are given by (39) and (40), respectively. Since a_2 is positive (from (40)), the proposition follows. $\|$.

Proof of Proposition 7: Note that $H_1 > 0$ and $k < -1$ imply that $\mathrm{corr}(F, P) < 0$ (from Proposition 3). Also observe from (25) that if $k > -(1+q)$, $\mathrm{corr}(T, P) > 0$. Then, we have from (38) and (6) that

$$\mathrm{cov}(\mu, P) = a_1 H_1 v_\theta + a_2 H_2 v_z. \tag{44}$$

From (39), (40), and (7) the right-hand side of (44) equals

$$\frac{m v_\theta [k m v_\theta (m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z) + \{m v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta)\}\{m^2 v_\theta + R^2 v_\epsilon^2 v_z\}]}{(m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z)(m^2 v_\theta + R^2 v_\epsilon^2 v_z)},$$

which is positive if $H_1 > 0$ (from (7)). The proposition thus follows. $\|$

Proof of Proposition 8: Now, it follows from (25) and (10) that

$$\beta_T = \frac{m v_\theta (1 + k + q)(m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z}{k m v_\theta (m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z) + [m v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta)](m^2 v_\theta + R^2 v_\epsilon^2 v_z)}. \tag{45}$$

Now, let

$$\begin{aligned} L \equiv\ & \Big[k m v_\theta (m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z) + m^3 v_\theta^2 + m^2 R^2 v_\epsilon v_\theta v_z (v_\epsilon + v_\theta) \\ + \ & m R^2 v_\epsilon^2 v_\theta v_z + R^4 v_\epsilon^3 v_z^2 (v_\epsilon + v_\theta)\Big]^2 . \end{aligned}$$

We then have

$$\begin{aligned} \frac{d\beta_T}{dv_z} =\ & -m R^2 v_\epsilon^2 v_\theta (1 + k + q) \Big[m^4 v_\theta^2 + 2 m^2 R^2 v_\epsilon v_\theta v_z (v_\epsilon + v_\theta) \\ + \ & m R^4 v_\epsilon^2 v_\theta v_z^2 (v_\epsilon + v_\theta) + R^4 v_\epsilon^3 v_z^2 (v_\epsilon + v_\theta)\Big] / L, \\ \frac{d\beta_T}{dv_\epsilon} =\ & -m R^2 v_\epsilon^2 v_\theta v_z (1 + k + q) \Big[2 m^4 v_\theta^2 + m^3 R^2 v_\epsilon v_\theta^2 v_z + m^2 R^2 v_\epsilon v_\theta v_z (4 v_\epsilon + 3 v_\theta) \end{aligned}$$

$$
\begin{aligned}
&+ \quad mR^4 v_\epsilon^2 v_\theta v_z^2 (3v_\epsilon + 2v_\theta) + R^4 v_\epsilon^3 v_z^2 (2v_\epsilon + v_\theta)\Big] /L, \text{ and} \\
\frac{d\beta_T}{dm} &= R^2 v_\epsilon^2 v_\theta v_z (1+k+q)\Big[m^4 v_\theta^2 + m^2 R^2 v_\epsilon v_\theta v_z (2v_\epsilon + 3v_\theta) \\
&+ \quad 2mR^4 v_\epsilon^2 v_\theta v_z^2 (v_\epsilon + v_\theta) + R^4 v_\epsilon^3 v_z^2 (v_\epsilon + v_\theta)\Big] /L.
\end{aligned}
$$

Since $L > 0$, and, under the conditions in Proposition 7, $1 + k + q > 0$, the first two derivatives are negative, and the third is positive. Also, note that $\beta_T > 0$, since the denominator of the right-hand side of (45) is identical to the denominator of (7), which is required to be positive for $H_1 > 0$ (a positive H_1 is a premise of Proposition 7).

We now turn to β_K. Let

$$\Delta \equiv \frac{C_3 + D_3 + C_3 D_4 + C_4 D_3}{1 - C_4 D_4}.$$

From (17) and (18), and noting that $P = H_1\theta + H_2 v_z$, and that $\mu = a_1\theta + a_2 z$, we have that

$$\beta_K = \frac{\Delta(a_1 H_1 v_\theta + a_2 H_2 v_z)}{H_1^2 v_\theta + H_2^2 v_z}.$$

Now, observing from Proposition 1 that $H_2 = (Rv_\epsilon/m)H_1$, and substituting for H_1, a_1, and a_2 from (7), (39), and (40), respectively, we find that

$$\beta_K = \frac{\beta_T \Delta}{(1+k+q)}.$$

Now, $\Delta > 0$, and, as noted above, the conditions in Proposition 7 imply that $1+k+q > 0$, indicating that $\beta_K > 0$. Further, Δ and $1 + k + q$ do not involve v_z, v_ϵ, or m. So, the signs of the derivatives of β_K with respect to v_z, v_ϵ, and m, are the same as the corresponding ones for β_T. This proves the proposition. $\|$

Proof of Proposition 9: Note that (19) can be written as

$$\begin{aligned}\pi_1 &= C_1 + \theta C_2 + (C_3 + C_4 D_3)\left[\frac{(C_3 + C_4 D_3)[2\mu\theta C_3(1 - C_4 D_4)}{2(1 - C_4 D_4)^2}\right. \\ &+ \left.\frac{\mu^2\{C_4 D_3 + C_3(2C_4 D_4 - 1)\}]}{2(1 - C_4 D_4)^2}\right]. \qquad (46)\end{aligned}$$

Now observe that $\mu = a_1\theta + a_2 z$, and from (39) and (40), a_1 and a_2 are positive. For an arbitrarily small shock $z = z_c$, the term involving $\mu^2 = a_1^2\theta^2 + a_2^2 z_c^2$ can be ignored; further, the coefficient of $\mu\theta = a_1\theta^2 + a_2\theta z_c$ in (46) is positive. Thus, a small shock $z = z_c > 0$ increases the right-hand side of (46) relative to the case where $z = 0$. $\|$

Proof of Proposition 10: Solving for the price P from (32), we have

$$P = \frac{RR_U v v_\epsilon z + k\mu[Rv_\epsilon - m(Rv_\epsilon + R_U v)] + m(R_U\theta v - \mu R v_\epsilon) + \mu R v_\epsilon}{Rv_\epsilon - m(Rv_\epsilon - R_U v)}. \qquad (47)$$

Note that since v and τ do not depend on the risk aversion of the uninformed, they remain unchanged (as does μ) relative to their algebraic representations in Section 2 and the proof of Proposition 1. Substituting for v and μ from (37) and (38), respectively, into (47) above yields (33). $\|$

Proof of Proposition 11: Under overconfidence, the variable $\tau = \theta + Rv_c z/m$. Thus, the analogs of (39) and (40) become

$$a_1 = \frac{m^2 v_\theta}{m^2 v_\theta + R^2 v_c^2 v_z} \qquad (48)$$

and

$$a_2 = \frac{Rmv_\theta v_c}{m^2 v_\theta + R^2 v_c^2 v_z}. \qquad (49)$$

Using techniques similar to that used for Proposition 1, we find that the price P takes the firm

$$P = H_1^o\theta + H_2^o z$$

where $H_1^o = A_o/B_o$, with

$$\begin{aligned} A_o &\equiv m(kmv_\theta(m^3v_\theta(Rv_c - R_Uv_\epsilon) - m^2Rv_cv_\theta + mR^2v_\epsilon v_z(Rv_c - R_U(v_\epsilon + v_\theta)) \\ &- R^3v_cv_\epsilon^2v_z) + m^4v_\theta^2(Rv_c - R_Uv_\epsilon) - m^3Rv_cv_\theta^2 + m^2R^2v_\epsilon v_\theta v_z(Rv_cv_\epsilon \\ &- R_U(v_C^2 + v_\epsilon(v_\epsilon + v_\theta))) - mR^3v_Cv_\epsilon^2v_\theta v_z - R^4R_Uv_C^2v_\epsilon^2v_z^2(v_\epsilon + v_\theta)) \end{aligned}$$

and

$$B_o \equiv (m^2v_\theta + R^2v_c^2v_z)(m^3v_\theta(Rv_c - R_Uv_\epsilon) - m^2Rv_cv_\theta + mR^2v_\epsilon v_z(Rv_c - R_U(v_\epsilon + v_\theta)) - R^3v_cv_\epsilon^2v_z).$$

Further, $H_2^o = (Rv_cH_1^o)/m$. Note that the covariance

$$\text{cov}(F, P) = H_1^o(1 + ka_1)v_\theta + H_2^oka_2v_z = H_1^o(1 + k)v_\theta.$$

We also have that

$$\text{var}(P) = H_1^{o2}v_\theta + H_2^{o2}v_z$$

and

$$\text{var}(F) = (1 + ka_1)^2v_\theta + v_\epsilon + k^2a_2^2v_z.$$

All this implies that

$$\text{corr}(F, P) = \frac{m\ \text{sgn}(H_1^o)(1 + k)v_\theta}{[k^2m^2v_\theta^2 + 2km^2v_\theta^2 + (v_\epsilon + v_\theta)(m^2v_\theta + R^2v_c^2v_\theta)]^{0.5}}.$$

The absolute magnitude of the above correlation increases as v_c decreases, because the absolute value of the numerator does not involve v_c and the denominator is increasing in v_c. ‖

Proof of Proposition 12: Suppose that the true variance of participation shocks is v_z' whereas uninformed agents estimate it to be v_z. The volatility of the price is

$$\text{var}_I(P) = H_1^2v_\theta + H_2^2v_z',$$

and when the true volatility of z is v_z, the volatility of the price is

$$\mathrm{var}(P) = H_1^2 v_\theta + H_2^2 v_z',$$

where the subscripts I denotes irrationality. Now, substituting for H_1 from (33), and noting from Proposition 10 that $H_2 = Rv_\epsilon H_1/m$, we have

$$\mathrm{var}_I(P) - \mathrm{var}(P) = \gamma_1/\gamma_2,$$

where

$$\begin{aligned}\gamma_1 \;\equiv\;& R^2 v_\epsilon^2 m^2 [kmv_\theta\{m^3 v_\theta(R - R_U) - m^2 R v_\theta + mR^2 v_\epsilon v_z(Rv_\epsilon - R_U(v_\epsilon + v_\theta)) - R^3 v_\epsilon^2 v_z\} \\ +\;& (m^2 v_\theta + R^2 v_\epsilon^2 v_z)\{m^2 v_\theta(R - R_U) - mRv_\theta - R^2 R_U v_\epsilon v_z(v_\epsilon + v_\theta)\}]^2\end{aligned}$$

and

$$\begin{aligned}\gamma_2 \;\equiv\;& m^2(m^2 v_\theta + R^2 v_\epsilon^2 v_z)^2 [m^3 v_\theta(R - R_U) - m^2 R v_\theta + mR^2 v_\epsilon v_z\{Rv_\epsilon - R_U(v_\epsilon + v_\theta)\} \\ -\;& R^3 v_\epsilon^2 v_z]^2.\end{aligned}$$

Since γ_1 and γ_2 are positive, the volatility of the price is therefore higher when the volatility of the participation shock is underestimated ($v_z < v_z'$). $\|$

Figure 1: Correlation between total cash flows and cash flows of the traded asset, as a function of the feedback parameter, k

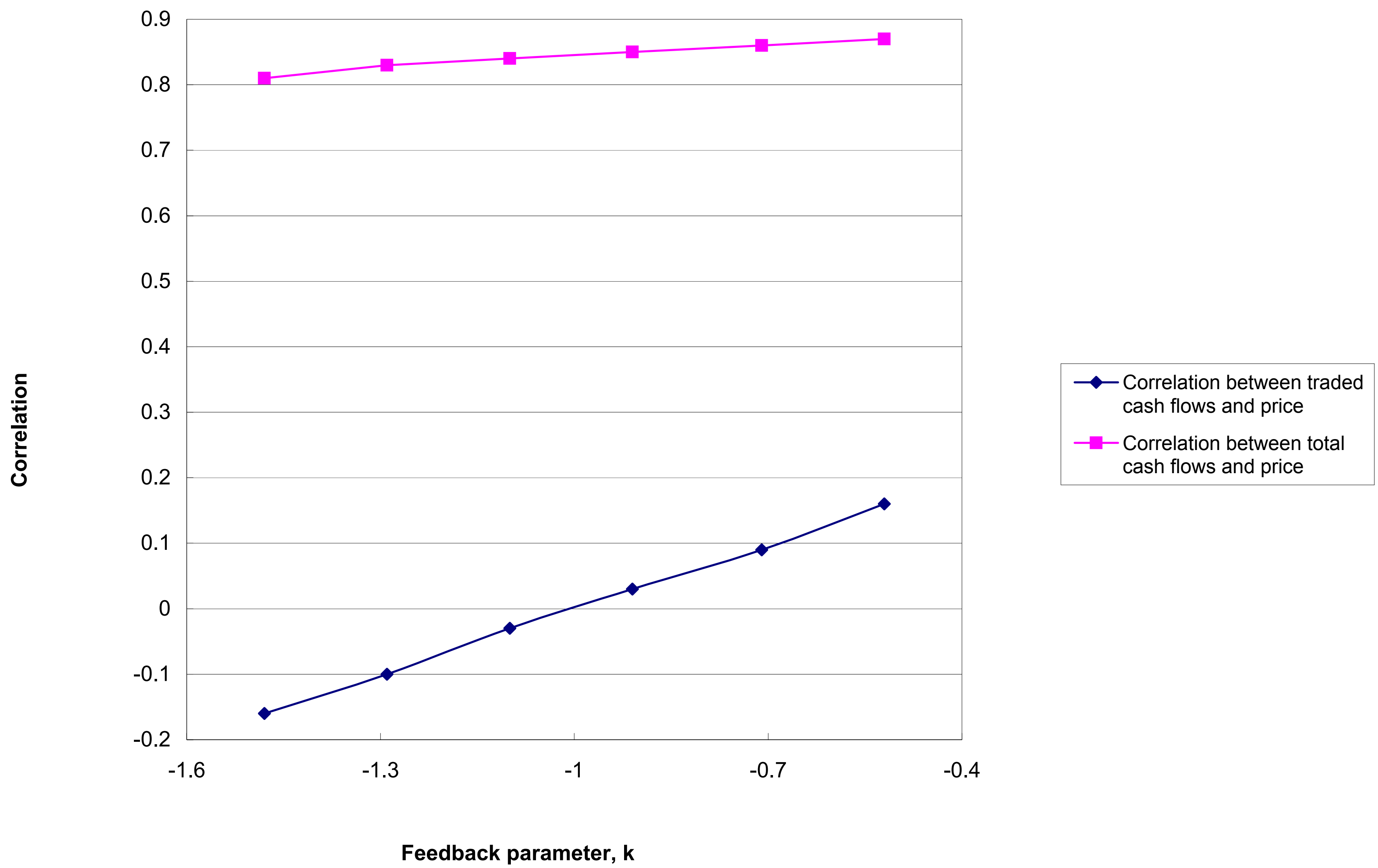

Figure 2: Correlation between cash flows of the traded asset and the market price as a function of the risk aversion of the informed agents (R) relative to that of the uninformed (R_U)

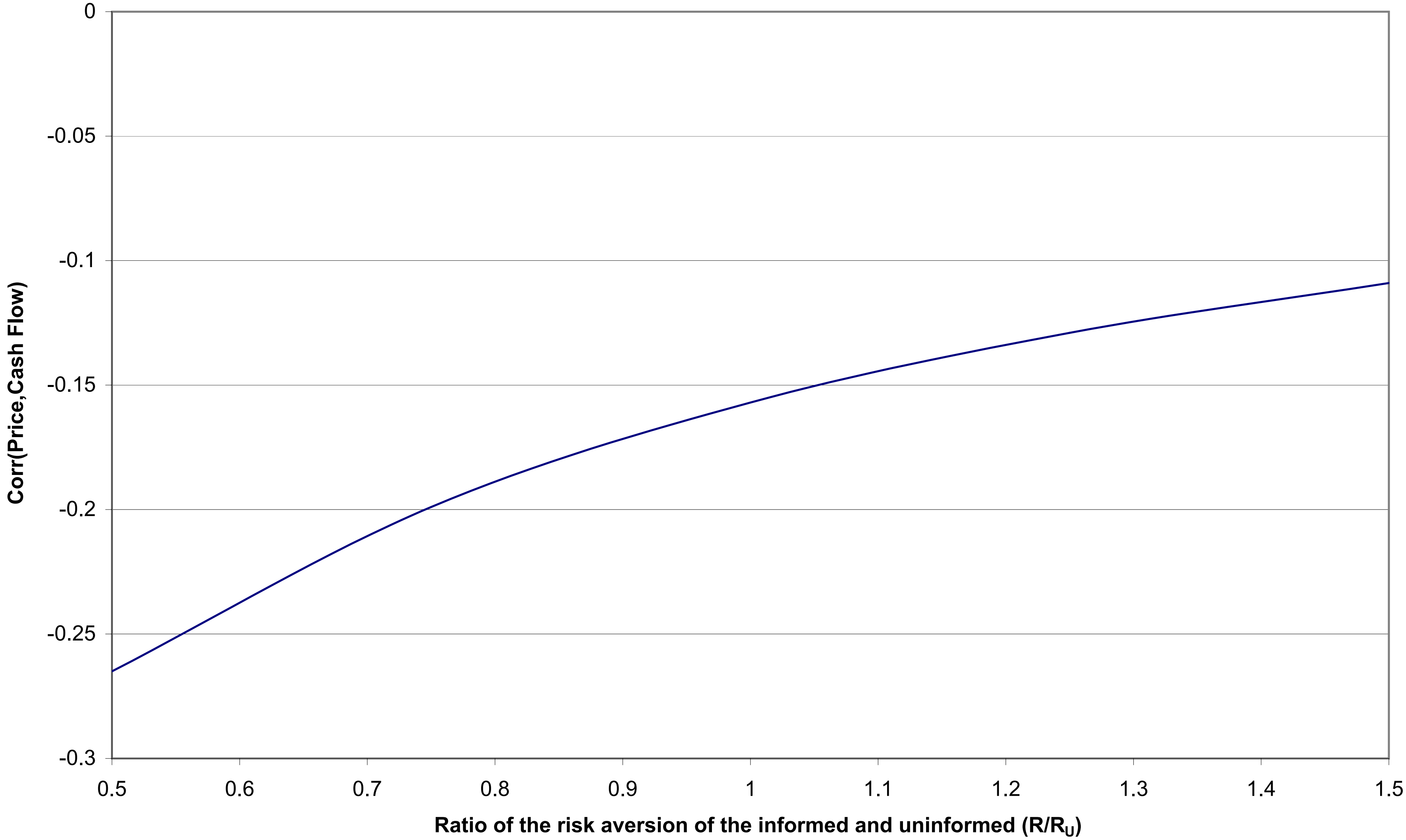

Figure 3: Correlation between cash flows of the traded asset and the market price as a function of the risk borne by the informed agents

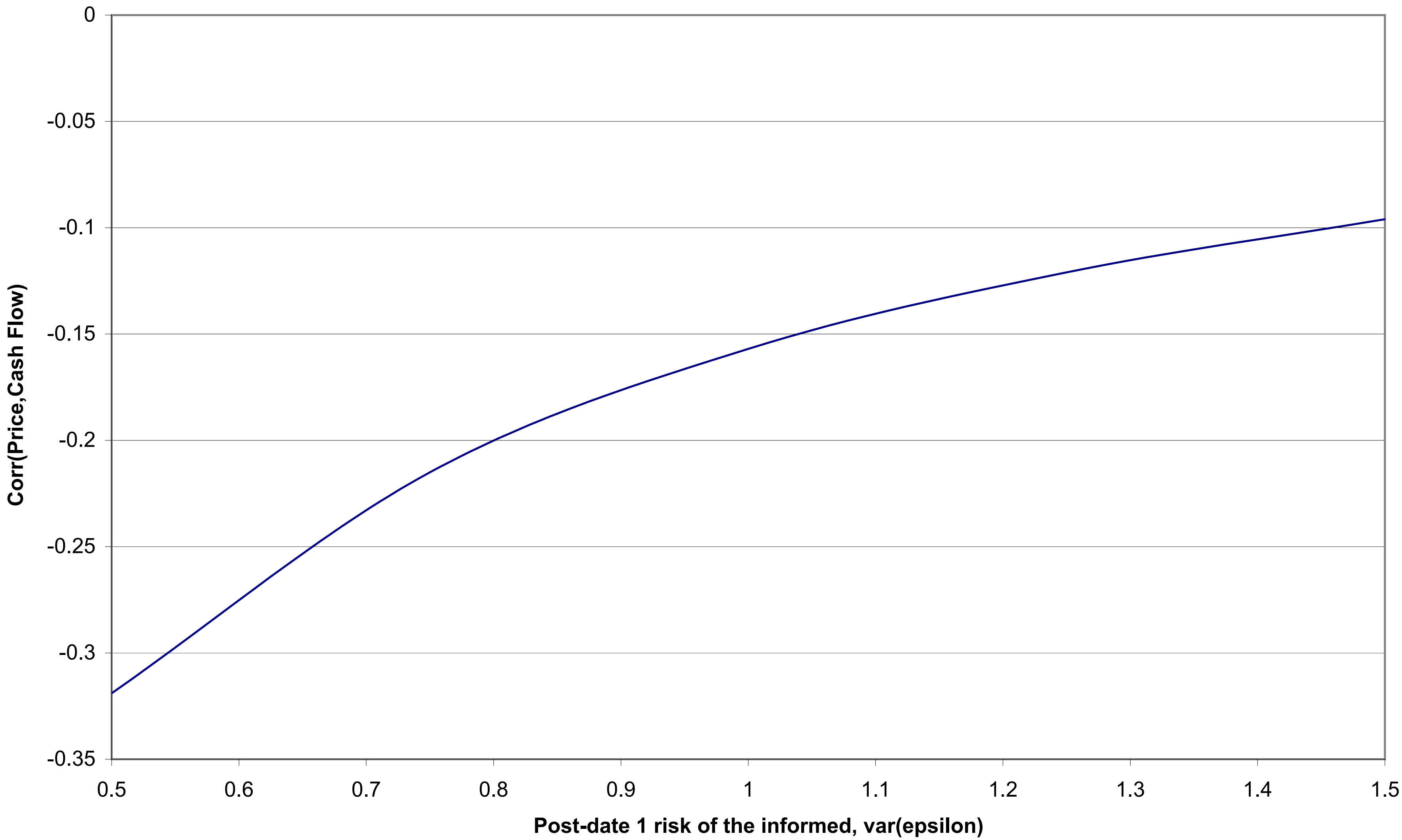

Figure 4: Correlation between cash flows of the traded firm and the market price as a function of the variance of the participation shock, v_z