Vertical Integration, Foreclosure, and Upstream Competition*

Job Market Paper

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Abstract

We develop a model, in which two vertically integrated firms compete, first, on an upstream market to supply an intermediate input to a downstream firm, and second, on a downstream market with the same downstream firm. We show that, even if firms compete in prices with homogenous products on the upstream market, the input may be priced above marginal cost in equilibrium. These partial foreclosure outcomes are more likely to arise when final products are close substitutes, when the downstream firm is relatively inefficient, or when integrated firms offer two-part tariffs on the upstream market. We show that these equilibria degrade both social welfare and consumers' surplus, relative to the Bertrand outcome, and we derive conditions under which an input price cap can restore the competitiveness of the upstream market. Performing comparative statics on the market structure, we find that an increase in the number of integrated or downstream firms can actually increase the scope for partial foreclosure equilibria. We then wonder whether situations, in which the downstream firm does not receive the input at all, can emerge in equilibrium. We show that such complete foreclosure equilibria are more likely to arise when downstream products are close substitutes, the downstream firm is relatively inefficient, and the input is poorly differentiated. Again, an increase in the number of integrated firms can make complete foreclosure more likely. Finally, we derive several results on the profitability and social desirability of horizontal and vertical mergers, with and without efficiency gains.

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1 Introduction

The anticompetitive effects of vertical mergers have long been a hotly debated issue among economists. Until the end of the 1960s, the traditional vertical foreclosure theory was widely accepted by antitrust practitioners. According to this theory, vertical mergers were harmful to competition, since vertically integrated firms had incentives to raise their rivals' costs. This view was seriously challenged by Chicago school authors in the 1970s, notably Bork (1978) and Posner (1976), on the ground that firms cannot leverage market power from one market to another. A more recent strategic approach of the subject, initiated by Ordover, Saloner, and Salop (1990) and Hart and Tirole (1990), shows how vertical integration might relax competition.

Remarkably, a substantial part of the literature has built around a common framework, introduced by Ordover, Saloner, and Salop (1990).[1] There are initially two identical unintegrated upstream firms, and two identical unintegrated downstream firms. The intermediate input, sold on the upstream market, is homogenous, while the final product, sold on the downstream market, is differentiated. In the first stage, downstream firms bid to acquire the first upstream firm. Then, if a merger has taken place, the remaining unintegrated downstream firm can bid to integrate backward with the remaining upstream firm. Upstream and downstream price competition take place in stages 3 and 4 respectively.

As pointed out by Chicago school authors, within this simple framework, a vertical merger cannot have anticompetitive effects. If no merger has taken place, then, firms compete fiercely on the upstream market, and the input ends up being priced at marginal cost. If one merger has taken place, one may be tempted to believe, as the traditional foreclosure proponents did, that the upstream price charged to the remaining unintegrated downstream firm should exceed marginal cost. However, as Chicago authors put it, if it were the case, the firm which does not supply the upstream market, be it integrated or not, would have a clear incentive to undercut the offer of its rival, and capture the upstream market. Therefore, even after a vertical merger, upstream competition leads to the Bertrand outcome.

This reasoning suggests that, even though integrated firms have incentives to raise their non-integrated rivals' costs, this does not annihilate the competitive pressure on the input market, and additional ingredients are needed to obtain anticompetitive effects from vertical integration within the common framework. Additional ingredients emphasized by the literature include an extra commitment power for vertically integrated firms on the upstream market (Ordover, Saloner, and Salop (1990)),[2] the choice of input specification (Choi and Yi (2000), Avenel and Barlet (2000)), upstream switching costs (Chen (2001)), tacit collusion (Nocke and White (2007), Normann (2009)) and exclusive dealing contracts (Chen and Riordan (2007)).

It is worth noting that, within the common framework, we never observe competition between vertically integrated firms on the input market. This is because if two vertical mergers take place in

[1] Another strand of the literature, initiated by Hart and Tirole (1990), focuses on the opportunism problem faced by an upstream bottleneck owner. See O'Brien and Shaffer (1992), McAfee and Schwartz (1994), Marx and Shaffer (2004), Rey and Vergé (2004), and Rey and Tirole (2005) for a survey.

[2] See Hart and Tirole (1990) and Reiffen (1992) for a critical view on this assumption.

Ordover, Saloner, and Salop (1990)'s model, there is no remaining unintegrated downstream firm. In this case, both integrated firms produce the input in-house, and the upstream market disappears. In this paper, we argue that analyzing competition between integrated firms yields interesting results both for practitioners and for theorists. From a theoretical point of view, we will see that upstream competition between integrated firms is less intense than competition between upstream firms, or than competition between upstream and integrated firms. In particular, in the model we develop, there can exist equilibria, in which an integrated firm sells the input at its monopoly price, while its integrated rival decides rationally to make no upstream offer. From a more applied point of view, we argue below that market structures in which the input market is essentially populated with integrated firms are commonly observed in several industries.

In the broadband market, Digital Subscriber Line (DSL) operators and cable networks own a broadband infrastructure and compete at the retail level. They can also compete to provide wholesale broadband services to unintegrated downstream firms, which have not built their own network. Similarly, in the mobile telephony market, Mobile Virtual Network Operators (MVNOs) do not have a spectrum license nor a mobile network and therefore have to purchase a wholesale mobile service from Mobile Network Operators (MNOs). Other examples can be found in licensing contexts. For instance, Arora, Fosfuri, and Gambardella (2001) report that, at the end of the 1990s, Dow Chemicals and Exxon had developed rival metallocene technologies, which enabled them to produce polyethylenes. They also licensed their technologies to downstream polyethylenes producers.[3] In the video game industry, some firms (e.g., Epic Games, Valve Corporation) have designed their own 3D engines to develop 3D video games. They also license these engines to rival downstream firms (e.g., Electronic Arts).

Such a market structure can also emerge endogenously. According to Riordan (2008), before 2001, the molded door market was organized as follows. Masonite and a vertically integrated firm, referred to as the non-party firm, produced molded doorskins. These doorksins were subsequently transformed into molded doors by downstream firms: Premdor, the non-party firm, and a competitive fringe of downstream producers. In 2001, Premdor made an offer to acquire Masonite. The U.S. Department of Justice eventually gave clearance to this merger, but forced Masonite to divest one of its plants to a new upstream entrant. More recently, a wave of vertical mergers took place in the digital map industry. Before 2007, there were essentially two digital map producers: Tele Atlas and NAVTEQ. These firms sold their maps to downstream personal navigation device producers, such as TomTom or Garmin, and to mobile phone manufacturers, such as Nokia or Siemens. At the end of 2007, TomTom notified the European Commission of its acquisition of Tele Atlas. Shortly after, Nokia reacted by notifying the Commission of its acquisition of NAVTEQ.[4] The Commission eventually cleared both mergers without conditions, and the digital maps market is now supplied by a duopoly of vertically integrated firms.

In Section 2, we develop a model, in which two vertically integrated firms and an unintegrated downstream firm compete in prices with differentiated products on a downstream market. The

[3]See Arora (1997) for other examples in the chemicals industry.
[4]See European Commission (2008).

goods sold to final consumers are derived from an intermediate input that the integrated firms can produce in-house. Integrated firms compete, first on the upstream market to provide the input to the unintegrated downstream firm, and second on the downstream market with the unintegrated downstream firm. We assume that the unintegrated downstream firm can also purchase the input from an inefficient alternative source. This assumption enables us to rule out complete foreclosure of the downstream entrant in equilibrium.[5] We relax it in Section 6. The upstream market exhibits the usual ingredients of tough competition: integrated firms compete in (linear) prices, produce a perfectly homogeneous upstream good, incur the same constant marginal cost, and all the assumptions that traditionally lead to equilibrium foreclosure in the literature are assumed away. Yet, we show in Section 3 that upstream competition may not drive the input price down to marginal cost, thereby giving rise to partial foreclosure equilibria. In particular, there can exist monopoly-like equilibria, in which one vertically integrated firm supplies the intermediate input at its monopoly upstream price, while its integrated rival makes no upstream offer.

The intuition is the following. Assume that integrated firm i supplies the wholesale market at a strictly positive price-cost margin, and consider the incentives of its integrated rival j to corner that market. Notice first that, when firm i increases its downstream price, it recognizes that some of the final consumers it loses will eventually purchase from the unintegrated downstream firm, thereby increasing upstream demand and revenues. This implies that firm i charges a higher downstream price than its integrated rival j at the downstream equilibrium. This effect obviously benefits firm j, which faces a less aggressive competitor on the final market: this is the softening effect. Now, if firm j undercuts firm i on the upstream market and becomes the upstream supplier, the roles are reversed: firm i decreases its downstream price, while firm j increases it. To sum up, firm j faces the following trade-off when deciding whether to undercut. On the one hand, undercutting yields upstream profits; on the other hand, it makes integrated firm i more aggressive on the downstream market. When the latter effect is strong enough, the incentives to undercut vanish and the Bertrand logic collapses.[6]

This implies that, when the softening effect is strong enough, the monopoly outcome on the upstream market may persist even under the threat of competition on that market. Other equilibria may exist, but monopoly-like equilibria are Pareto-dominant from the integrated firms' viewpoint.

Two factors are shown to have an important impact on the tradeoff between the softening effect and the upstream profit effect. First, the degree of differentiation at the downstream level has a direct impact on the strength of the softening effect. Intuitively, when final products are strongly differentiated, downstream demands are almost independent and the softening effect is consequently weak. As a result, undercutting on the upstream market is always profitable, and competition drives the wholesale price down to marginal cost. Conversely, when downstream products are

[5]In this paper, we distinguish two types of foreclosure. By complete foreclosure, we mean that the downstream firm does not manage to obtain the input, and is therefore excluded from the downstream market. Partial foreclosure, on the other hand, means that the entrant receives the input at a price above marginal cost.

[6]That an integrated firm changes its downstream behavior when it supplies a non-integrated rival has already been noted in the literature. See Chen (2001), Fauli-Oller and Sandonis (2002), Sappington (2005) and Chen and Riordan (2007) among others. The novelty of our paper is to analyze the implications of these upstream-downstream interactions on upstream competition between vertically integrated firms.

strong substitutes, the softening effect is strong and the monopoly outcome therefore emerges in equilibrium. Second, the downstream firm's cost (dis-)advantage affects directly the strength of the upstream profit effect. When firm d is relatively inefficient relative to integrated firms, its input demand and therefore the upstream profits are low. This reduces the incentives to undercut the upstream market, and therefore makes foreclosure more likely.

We obtain even stronger results under two-part tariff competition. We show that partial foreclosure equilibria with strictly positive upstream profits always exist when firms compete in two-part tariffs on the upstream market. In equilibrium, the upstream supplier sets the variable part that maximize its joint profit with the downstream firm, and adjusts the fixed part to ensure that the downstream firm makes non-negative profits, and that the other integrated firm does not want to undercut.

In Section 4, we show that partial foreclosure equilibria degrade both social welfare and consumers' surplus. We show that a price cap over integrated firms' upstream offers can be an efficient means to regulate the wholesale market. More precisely, under some technical conditions, a price cap may destroy all partial foreclosure equilibria, even though the price cap does not bind in equilibrium.

In Section 5, we investigate the robustness of our results to changes in the market structure. We derive some comparative statics on the impact of the number of integrated firms or downstream firms, and of the mix between integrated firms and downstream firms, on the emergence of equilibrium foreclosure. Conventional wisdom would suggest that, say, an increase in the number of integrated firms should intensify upstream competition, and therefore make partial foreclosure a less likely outcome. This reasoning seems attractive, but it fails to account for the strong interactions between the upstream and the downstream market. When the number of integrated firms increases, downstream competition becomes tougher, which lowers the input demand of downstream firms, and therefore the upstream profits. At the same time, the softening effect becomes less important, since when the upstream supplier increases its downstream price, a lower fraction of downstream consumers switch to the downstream firms' products. The overall impact is therefore ambiguous. We claim that changes in the number of downstream firms, or in the mix between integrated and downstream firms, also move the upstream profit effect and the softening effect in the same direction, so that nothing ensures that these changes will make foreclosure less likely. To sort out these effects, we solve the model under a linear specification of downstream demands, and show that the number of integrated firms has a non-monotonic impact on the likelihood of partial foreclosure, whereas more downstream firms tends to make foreclosure more likely.

In Section 6, we relax the assumption that the downstream firm can purchase the input from an alternative source when it receives no upstream offers from the integrated firms, which raises the issue of complete foreclosure. As a first step, we remove firm 2 from the industry, so that firm 1 owns an upstream bottleneck. When firm 1 considers whether to supply the input to firm d, it trades off two effects: the upstream profit effect, and the cannibalization effect. We show that, as long as firm d is not too inefficient, complete foreclosure does not arise, since firm 1 prefers to use the downstream entrant to reach new final consumers.

When integrated firm 2 is present, the tradeoff faced by firm 1 when deciding whether to supply the entrant is modified as follows. In addition to the upstream profit and the cannibalization effects, firm 1 must also take into account the softening effect, which we presented above, and the reaction effect, according to which firm 2 will react to the entry of firm d by pricing more aggressively on the final market. Because of the adverse impact of the reaction effect on the upstream supplier's profit, it may then be that firm 1 prefers to foreclose the entrant completely, even if firm d is as efficient as its integrated rivals. This typically happens when final products are close substitutes. As was the case for partial foreclosure, having more integrated firms may therefore not promote upstream competition, but instead trigger the emergence of complete foreclosure in equilibrium. We also perform comparative statics on other parameters, and find that complete foreclosure is less likely when firm d is relatively efficient, or when the input is differentiated.

We endogenize the market structure in Section 7. We assume that the industry is initially non-integrated, with three downstream firms and two upstream firms, and we allow downstream firms to bid to acquire upstream firms, as in Ordover, Saloner, and Salop (1990). We show that, even if vertical mergers do not create efficiency gains, there exists an equilibrium with two vertical mergers, when firms anticipate that they will be able to implement a partial foreclosure equilibrium in the two-merger subgame. In our model, vertical mergers can therefore arise for purely anticompetitive reasons. We then analyze the impact of efficiency gains, by assuming that a vertical merger reduces the upstream or downstream marginal costs of the merging parties. With efficiency gains, two mergers always take place in equilibrium. Since the second merger involves both an efficiency effect and a foreclosure effect, it not clear whether it should be given clearance by antitrust authorities. Common sense would suggest the following rule of thumb: the competition authority should be more favorable to the second merger when efficiency gains are larger. We show that this simple rule of thumb is indeed supported by our theory when efficiency gains reduce the upstream marginal cost. By contrast, with downstream synergies, we show that stronger efficiency gains can actually increase the scope for partial foreclosure. In this case, a stronger efficiency effect may also trigger a foreclosure effect from the second merger, and the rule of thumb described before is not necessarily accurate.

We also analyze the impact of a horizontal merger between an integrated firm and a downstream firm, in our framework with two integrated firms and one downstream firm. We find that, contrary to what a single-market analysis would predict (see Deneckere and Davidson (1985)), a horizontal merger is not necessarily profitable, even though firms compete in prices with differentiated products on the final market. This is because a partial foreclosure equilibrium may be an efficient means to soften downstream competition, as it provides the upstream supplier and the downstream firm with a commitment not to compete aggressively. In particular, we show that, when integrated firms compete with two-part tariffs on the upstream market, and when final products are sufficiently close substitutes, a horizontal merger is not profitable. Interestingly, for intermediate values of the substitutability parameter, we find that a horizontal merger is profitable, and that it increases both social welfare and consumers' surplus, even if the merger does not involve any form of synergies. This result emphasizes the fact that it can be important to take into account the vertical dimensions of a horizontal merger when deciding whether to challenge it.

Our paper contributes to the literature on the anticompetitive effects of vertical mergers. As explained earlier on, this literature has developed around a common framework, in which, by construction, upstream competition between vertically integrated firms never arises. Our contribution is to show that competition between integrated firms can be quite soft. This implies that vertical mergers that lead to a situation in which the upstream market is essentially populated with integrated firms can have anticompetitive effects, absent all the elements which are known to generate foreclosure in the literature. The softening effect, which is key to understanding our foreclosure result, was first exhibited by Chen (2001).[7] Chen shows that when there is one vertical merger, the remaining downstream firm prefers purchasing the input from the integrated firm rather than buying it from the unintegrated upstream firm, in order to benefit from the softening effect. If there are upstream cost asymmetries and upstream switching costs, then the unintegrated upstream firm is unable to undercut the integrated firm on the upstream market and there is partial foreclosure in equilibrium. Our result is different. We show that when two integrated firms compete on the upstream market, the integrated rival is *able* to undercut, since we assume away cost differentials and switching costs, but it is not *willing* to do so. Our result also provides support to the classical analysis of Ordover, Saloner and Salop (1990), for we show that no commitment is actually necessary to sustain the monopoly outcome when the softening effect is strong enough.

A few papers do consider market structures with several integrated firms competing on the upstream market. Nocke and White (2007) investigate whether vertical mergers can facilitate upstream tacit collusion. In some circumstances, the upstream market ends up being populated with integrated firms only, but Nocke and White do not look for partial foreclosure equilibria in the one-shot game. Salinger (1988) develops a model of successive vertical oligopolies, in which firms compete *à la* Cournot on both markets. While this model enables him to derive interesting predictions for the impact of vertical integration on upstream and downstream markups, it has kind of a black-box flavor. In particular, as Riordan (2008) points out, it is unclear which assumptions should be made about the rationing rule on the upstream market, to obtain a result equivalent to Kreps and Scheinkman (1983) for successive vertical oligopolies models.

In a recent paper motivated by the mobile telephony industry, Ordover and Shaffer (2007) investigate the conditions under which an MVNO can be completely foreclosed by MNOs. Contrary to our results, they predict that an MVNO is more likely to be completely foreclosed when the input is differentiated. In Section 6, we argue that this result may be driven by the specific demands system they use, which fails to ensure consistency between the triopoly and duopoly demand functions. Höffler and Schmidt (2008) take a complementary perspective and study the impact on consumers' surplus of the entry of unintegrated downstream firms. They show that downstream entry can be detrimental to consumers, due to the softening effect. However, they assume away any form of wholesale competition: an upstream supplier is exogenously chosen, and it is free to impose its monopoly wholesale price. Our results indicate that allowing competition on the upstream market

[7] Chen (2001) refers to it as the collusive effect. We adopt a different terminology to make clear that our results do not involve any form of tacit or overt collusion.

may leave integrated firms with as much market power as when the upstream market structure is exogenously fixed.

2 The Model

Firms. There are two vertically integrated firms, denoted by 1 and 2, and one unintegrated downstream firm, denoted by d. Integrated firms are composed of an upstream and a downstream unit, which produce the intermediate input and the final good, respectively. The unintegrated downstream competitor is composed of a downstream unit only. In order to be active on the final market, it must purchase an intermediate input. Both integrated firms produce the upstream good under constant returns to scale at unit cost m. The downstream firm can either purchase the input from one of the integrated firms, or obtain it from an alternative source at a constant marginal cost $\overline{m} > m$.[8] The downstream product is derived from the intermediate input on a one-to-one basis with the twice continuously differentiable cost function $c_k(.)$, for firm $k \in \{1, 2, d\}$. We assume that integrated firms have the same downstream cost function: $c_1(.) = c_2(.)$.

Markets. All firms compete in prices on the downstream market and provide imperfect substitutes to final customers. Let p_k be the downstream price set by firm $k \in \{1, 2, d\}$ and $p \equiv (p_1, p_2, p_d)$ the vector of final prices. Firm k's demand, denoted by $q_k(p)$, is twice continuously differentiable; it depends negatively on firm k's price and positively on its competitors' prices: $\partial q_k/\partial p_k \leq 0$ with a strict inequality whenever $q_k > 0$, and $\partial q_k/\partial p_{k'} \geq 0$ with a strict inequality whenever $q_k > 0$ and $q_{k'} > 0$, for $k \neq k' \in \{1, 2, d\}$. We assume that demands have a finite choke point: for all $k = 1, 2, d$, for all p_{-k}, there exists $\overline{p_k}$ such that $q_k(\overline{p_k}, p_{-k}) = 0$.[9] We also suppose that the total demand is non-increasing in each price: for all $k' \in \{1, 2, d\}$, $\sum_{k\in\{1,2,d\}} \partial q_k/\partial p_{k'} \leq 0$. Symmetry of the integrated firms is assumed again: $q_1(p_1, p_2, p_d) = q_2(p_2, p_1, p_d)$ and $q_d(p_1, p_2, p_d) = q_d(p_2, p_1, p_d)$ for all p.

On the upstream market, integrated firms compete in prices and offer perfectly homogeneous products. We denote by w_i the upstream price set by integrated firm $i \in \{1, 2\}$.[10,11] The structure of the model is summarized in Figure 1.

Timing. The sequence of decision-making is as follows:

Stage 1 – Upstream competition: Vertically integrated firms announce their prices on the upstream market. Then, the unintegrated downstream firm elects at most one upstream provider.[12]

[8] This assumption is also made, e.g., by Ordover, Saloner, and Salop (1990) and Hart and Tirole (1990). It enables us to focus on situations in which the downstream firm is always active on the final market. We relax it in Section 6. The alternative source of supply can come from a competitive fringe of inefficient upstream firms.

[9] As usual, p_{-k} is the vector obtained by removing p_k from vector p.

[10] Throughout the paper, subscripts i and j refer to integrated firms only, whereas subscript k refers either to an integrated firm or to the unintegrated downstream firm.

[11] We analyze two-part tariff competition in Section 3.4.

[12] In Section 5, we show that our results would not be qualitatively affected if we allowed firm d to split its demand between the two integrated firms when it is indifferent between both offers.

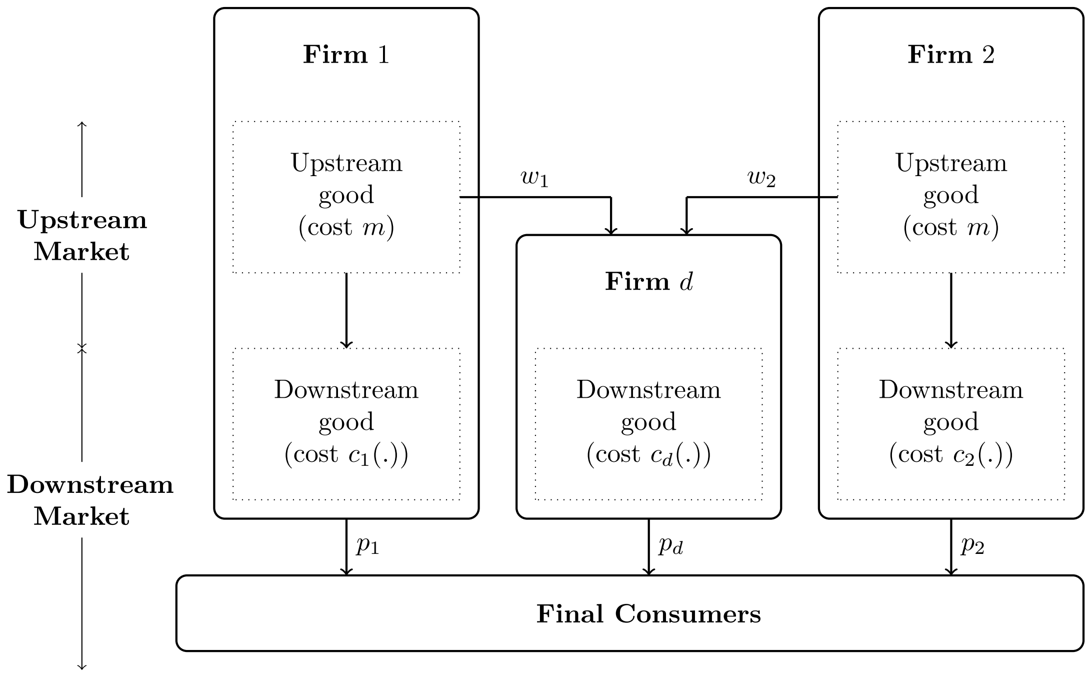


Figure 1: Structure of the model.

Stage 2 – Downstream competition: All firms set their prices on the downstream market. Then, the unintegrated downstream firm is allowed to switch to another upstream supplier, if this is strictly profitable.[13]

We focus on pure strategy subgame-perfect equilibria and reason by backward induction.

Profits. Assume first that the downstream firm purchases the input from integrated firm $i \in \{1,2\}$ at price w. The profit of firm i is given by:[14]

$$\tilde{\pi}_i^{(i)}(p,w) = (p_i - m)q_i(p) - c_i(q_i(p)) + (w - m)q_d(p).$$

The profit of integrated firm $j \neq i \in \{1,2\}$ which does not supply the upstream market is:

$$\tilde{\pi}_j^{(i)}(p,w) = (p_j - m)q_j(p) - c_j(q_j(p)).$$

The profit of unintegrated downstream firm d is:

$$\tilde{\pi}_d^{(i)}(p,w) = (p_d - w)q_d(p) - c_d(q_d(p)).$$

[13] Assuming that firm d can switch to another upstream supplier after downstream prices have been set simplifies the analysis by ensuring that the downstream firm always chooses the cheapest offer. This is in contrast to Chen (2001), in which upstream switching costs, together with an upstream cost asymmetry, generate anticompetitive vertical mergers. Our results would not be affected if we did not allow firm d to switch in stage 2, as long as the downstream firm's profit decreases in the input price.

[14] Throughout the paper, the superscript in parenthesis indicates the identity of the upstream supplier.

Note that when the upstream price is equal to the upstream unit cost, i.e., $w = m$, there is no upstream profit and all firms compete on a level playing field. This is the perfect competition outcome on the upstream market.

When the downstream firm obtains its input from the alternative source, the profit of integrated firm $i \in 1, 2$ can be written as:

$$\tilde{\pi}_i^{(\emptyset)}(p, \overline{m}) = (p_i - m)q_i(p) - c_i(q_i(p)),$$

and the profit of firm d is given by:

$$\tilde{\pi}_d^{(\emptyset)}(p, \overline{m}) = (p_d - \overline{m})q_d(p) - c_d(q_d(p)).$$

Downstream competition subgame. Since the upstream supplier is chosen after downstream prices have been set, it is clear that the downstream firm will purchase from the alternative source whenever both integrated firms' prices are above $\overline{m}$. This implies that we can restrict ourselves to situations in which firm d obtains the input at a price lower than or equal to $\overline{m}$. If an integrated firm sets a price above $\overline{m}$, we say that this firm makes no upstream offer, or that it offers $w = +\infty$.

For all pairs $(i,\ w)$, where $i \in \{1, 2, \emptyset\}$ denotes the upstream supplier, and $w \leq \overline{m}$ is the corresponding input price, we make the following assumptions:

(i) Firms' best responses on the downstream market are unique and well-defined by the corresponding first-order conditions: $\partial\tilde{\pi}_k^{(i)}/\partial p_k = 0$, for all $k \in \{1, 2, d\}$.

(ii) There exists a unique (pure-strategy) Nash equilibrium on the downstream market. We denote downstream equilibrium prices by $p_k^{(i)}(w)$, for $k = 1, 2, d$, and the corresponding price vector by $p^{(i)}(w)$.

(iii) Prices are strategic complements: for all $k \neq k'$ in $\{1, 2, d\}$, $\partial^2\tilde{\pi}_k^{(i)}/\partial p_k \partial p_{k'} > 0$.

Assumption (i) together with (iii) implies that the best response function of a firm is increasing in its rivals' prices. Combining (ii) with (iii), we also get that the unique downstream equilibrium is stable.[15] Finally, we assume that $\overline{m}$ is a relevant outside option: whatever the market structure, an unintegrated downstream firm earns strictly positive profits if it buys the intermediate input at a price lower than or equal to $\overline{m}$. In the following, we denote by $\pi_k^{(i)}(w)$ the profits earned by firm $k \in \{1, 2, d\}$ at the downstream equilibrium, when the input is supplied by $i \in \{1, 2, \emptyset\}$ at price $w \leq \overline{m}$.

Some preliminary results. Before moving to the main results, we derive three lemmas, which will prove useful in the following.

Lemma 1. *There are no equilibria in which the alternative source supplies the input to firm d.*

Proof. See Appendix A.2. □

[15] See Vives (1999), p.54.

The intuition behind Lemma 1 already exists in the previous literature (see, for instance, Chen (2001) and Fauli-Oller and Sandonis (2002)). In a nutshell, if the alternative source supplies the input at price $\overline{m}$, then, one integrated firm has an incentive to undercut this price. By doing so, it captures the upstream market, and it relaxes competition on the downstream market. This result is proven rigorously in Appendix A.2.

Lemma 2. *The perfect competition outcome ($w_1 = w_2 = m$) is always an equilibrium.*

Proof. See Appendix A.3. □

As conventional wisdom suggests, there always exists an equilibrium in which the input is priced at marginal cost. However, we will see in the following that other equilibria may exist, that are much less competitive.

For future references, let us define $w_m \equiv \arg\max_{w \leq \overline{m}} \pi_i^{(i)}(w)$ $(i = 1, 2)$, and assume that w_m is unique for simplicity. w_m is the monopoly upstream price, i.e., the price that integrated firm i would set if it were exogenously granted a monopoly position over the supply of input to firm d. We prove the following lemma:

Lemma 3. *$w_m > m$: monopoly power generates a positive markup on the input market.*

Proof. See Appendix A.4. □

In the following, we will see that this monopoly outcome can be sustained in a subgame-perfect equilibrium.

3 The Determinants of Partial Foreclosure

3.1 Partial Foreclosure and the Softening Effect

In this section, we show that the usual mechanism of Bertrand competition may be flawed on the upstream market, so that partial foreclosure equilibria may exist. Assume that integrated firm i has made an upstream offer to firm d, $m < w \leq \overline{m}$, and let us see whether integrated firm $j \neq i$ is willing to slightly undercut to corner the upstream market, as would be the case with standard (single-market) Bertrand competition.

The integrated firms' best-responses on the downstream market are characterized by the following first-order conditions:

$$\frac{\partial \tilde{\pi}_i^{(i)}}{\partial p_i}(p, w) = q_i + (p_i - c_i'(q_i) - m)\frac{\partial q_i}{\partial p_i} + (w - m)\frac{\partial q_d}{\partial p_i} = 0, \tag{1}$$

$$\frac{\partial \tilde{\pi}_j^{(i)}}{\partial p_j}(p, w) = q_j + (p_j - c_j'(q_j) - m)\frac{\partial q_j}{\partial p_j} = 0. \tag{2}$$

The comparison between (1) and (2) indicates that the upstream supplier has more incentives to raise its downstream price than its integrated rival. Realizing that final customers lost on the downstream market may be recovered via the upstream market, the upstream supplier is less aggressive than its integrated rival on the downstream market. Together with our stability assumption, this implies that the upstream supplier i ends up charging a higher downstream price than its integrated rival j at the subgame equilibrium: $p_i^{(i)}(w) > p_j^{(i)}(w)$. By symmetry between vertically integrated firms, this also implies that firm i charges a higher downstream price when it supplies the upstream market at price w, than when its integrated rival does: $p_i^{(i)}(w) > p_i^{(j)}(w)$. This is the *softening effect*.

As a result, following a straightforward revealed preference argument, firm j earns smaller downstream profits when it supplies the upstream market than when firm i does. These insights are summarized in the following lemma:

Proposition 1. *Let $m < w \leq \overline{m}$, and $i \neq j$ in $\{1, 2\}$. Then,*

$$p_i^{(i)}(w) > p_i^{(j)}(w), \tag{3}$$

$$(p_j^{(j)}(w) - m)q_j(p^{(j)}(w)) - c_j(q_j(p^{(j)}(w))) < (p_j^{(i)}(w) - m)q_j(p^{(i)}(w)) - c_j(q_j(p^{(i)}(w))). \tag{4}$$

Proof. See Appendix A.5. □

An important consequence of that result is that firm j may not necessarily want to undercut the input price, when firm i supplies the upstream market at $w > m$. If firm j undercuts, it captures the upstream profits, but, at the same time, firm i's downstream price decreases from $p_i^{(i)}(w)$ to $p_i^{(j)}(w)$, and firm j therefore faces tougher downstream competition. If the softening effect is strong enough, the incentives to undercut the input price vanish, and the Bertrand logic collapses. In particular, as shown in the following proposition, the monopoly outcome, and other partial foreclosure outcomes, may be equilibria:

Proposition 2. *There exists an equilibrium in which one integrated firm proposes w_m, and the other integrated makes no offer if, and only if,*

$$\pi_j^{(i)}(w_m) \geq \pi_i^{(i)}(w_m). \tag{5}$$

These equilibria are referred to as monopoly-like equilibria.
All other equilibria feature both vertically integrated firms setting the same input price, and earning the same profits.
From the integrated firms' point of view, when $\pi_j^{(i)}(w_m) > \pi_i^{(i)}(w_m)$,[16] *monopoly-like equilibria*

- *Pareto-dominate all other equilibria,*
- *Are the only equilibria involving no weakly dominated strategies.*

Proof. See Appendix A.6. □

When the softening effect is strong enough so that condition (5) holds, the hypothetical situation in which one of the integrated firm has exogenously exited the upstream market, granting a

[16]When $\pi_j^{(i)}(w_m) = \pi_i^{(i)}(w_m)$, both $(w_m, +\infty)$ and (w_m, w_m) can be sustained in equilibrium.

monopoly position to the other integrated firm, is an equilibrium. This might sound somewhat tautological. Yet, our contribution is to show that condition (5) may well be satisfied, because losers on the upstream market become winners on the downstream market.[17] Notice also that monopoly-like equilibria come by pairs since the upstream supplier can be either firm i or firm j.

Proposition 2 gives foundations to the classical analysis of Ordover, Saloner and Salop (1990), in which a vertically integrated firm commits to exiting the upstream market in order to let the upstream rival charge the monopoly price. We show that no commitment is actually necessary when the upstream rival is integrated, provided that the softening effect is strong enough.

All other equilibria feature both vertically integrated firms setting the same upstream price (and only one of them actually supplying the market). Such an outcome is part of an equilibrium only if the softening effect and the upstream upstream profit effect exactly cancel out, so that the upstream supplier earns as much profits as the vertically integrated firm which does not supply the upstream market. Formally: $\pi_i^{(i)}(w) = \pi_j^{(i)}(w)$. The Bertrand outcome is one such symmetric equilibrium. Other symmetric equilibria can also feature an upstream price strictly above m, as well as strictly below m.[18]

This multiplicity of equilibria can be resolved using standard selection criteria. First, the monopoly-like equilibria Pareto-dominate all the symmetric equilibria from the integrated firms' standpoint. Indeed, the upstream price in a symmetric equilibrium is always smaller than w_m, otherwise one of the vertically integrated firm would set w_m. This implies that, in the symmetric equilibria, both vertically integrated firms earn less than $\pi_i^{(i)}(w_m)$, which is lower than $\pi_j^{(i)}(w_m)$.

Second, the monopoly-like equilibria are the only equilibria involving no weakly dominated strategies. In particular, any symmetric equilibrium strategy is weakly dominated by w_m. Therefore, it seems reasonable to think that integrated firms will coordinate on one of the monopoly-like equilibria.

3.2 The Dilemma between Upstream and Downstream Competitiveness

A key determinant of the persistence of the monopoly outcome is the degree of differentiation of the unintegrated downstream firm. Suppose that the entrant is on a niche market, in the sense that its demand does not depend on the prices set by the downstream rivals and vice-versa.[19] In that situation, the wholesale profit of the upstream supplier is fully disconnected from its retail behavior and the softening effect disappears. Hence, with an unintegrated downstream firm on a niche market, the perfect competition outcome always emerges in equilibrium.

In order to refine this intuition, consider the symmetric linear case. The demand that addresses to firm $k \in \{1, 2, d\}$ is given by $q_k(p) = D - p_k - \gamma(p_k - \frac{p_1+p_2+p_d}{3})$, where $\gamma \geq 0$ parameterizes the degree of differentiation between final products, which can be interpreted as the intensity of downstream competition. Perfect competition corresponds to γ approaching infinity and local

[17] In the following sections, we provide several examples, with standard demand functions, in which condition 5 is indeed satisfied.

[18] When $w < m$, upstream profits are negative and the softening effect is reversed, with the upstream supplier adopting an aggressive stance on the downstream market to limit its upstream losses.

[19] Formally, $\partial q_d/\partial p_i = \partial q_i/\partial p_d = 0$ for $i \in \{1, 2\}$.

monopolies to $\gamma = 0$. All firms incur the same linear downstream costs: $c_k(q) = cq$.[20] With that specification, the assumptions we have made on the second stage demands, payoff functions, best-responses, etc. are satisfied.

Figure 2 offers a graphical representation of the profit functions $\pi_i^{(i)}(.)$, $\pi_j^{(i)}(.)$ and $\pi_d^{(i)}(.)$. As discussed in Section 3.2, when $w_i > m$, two opposite effects are at work. On the one hand, the upstream supplier derives profit from the upstream market; on the other hand, its integrated rival benefits from the softening effect on the downstream market. When the upstream price is not too high, the upstream profit effect dominates and $\pi_i^{(i)}(w_i) > \pi_j^{(i)}(w_i)$. When the upstream price is high enough, upstream revenues shrink, the softening effect is strengthened and $\pi_i^{(i)}(w_i) < \pi_j^{(i)}(w_i)$.

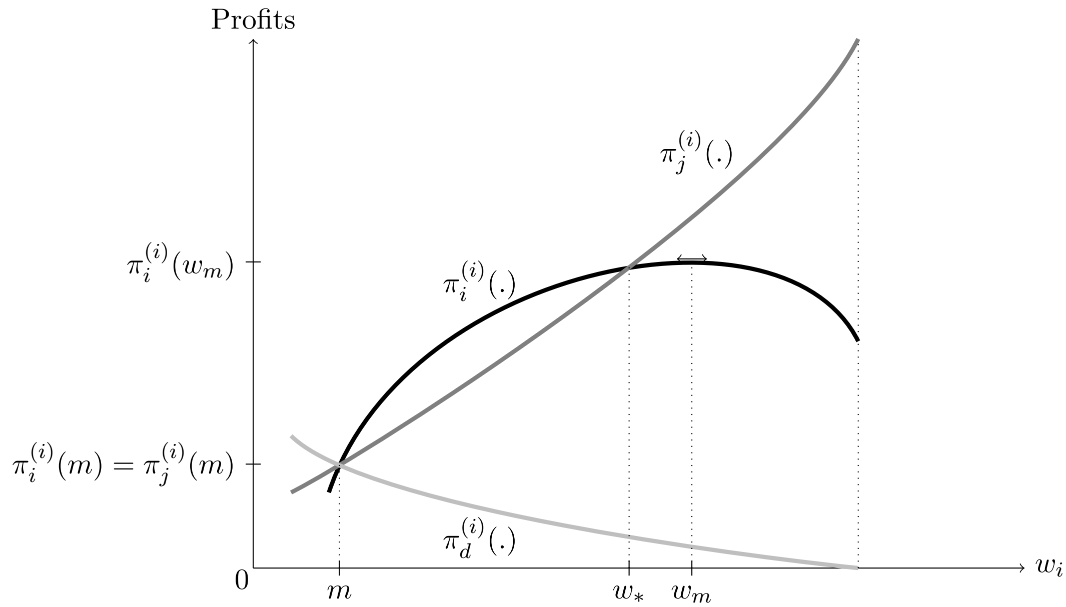


Figure 2: Profits in the symmetric linear case ($\gamma \geq \bar{\gamma}$).

We then obtain the following proposition.[21]

Proposition 3. *Consider the symmetric linear case. There exists $\bar{\gamma} > 0$ such that:*
If $\gamma \geq \bar{\gamma}$, then there exist four equilibrium outcomes on the upstream market:[22]

- *the* perfect competition *outcome;*
- *a supra-competitive* symmetric *outcome;*
- *two* monopoly-like *outcomes.*

[20]We assume that the total unit cost $m + c$ is strictly smaller than the intercept of the demand functions D, otherwise, it would not be profitable to be active on the final market.

[21]To derive this proposition, we assume that $\overline{m}$ is sufficiently high, so that the constraint $w \leq \overline{m}$ does not bind in maximization problem $\max_{w \leq \overline{m}} \pi_i^{(i)}(w)$. In the symmetric linear case, this interior w_m is always such that firm d remains active on the downstream market.

[22]The perfect competition and monopoly-like equilibria are stable; the supra-competitive symmetric equilibrium is unstable.

Otherwise, the perfect competition outcome is the only equilibrium outcome.

Proof. See Appendix A.7. □

To grasp the intuition of the proposition, suppose that the upstream market is supplied at the monopoly upstream price. When the substitutability between final products is strong, the integrated firm which supplies the upstream market is reluctant to set too low a downstream price since this would strongly contract its upstream profit. The other integrated firm benefits from a substantial softening effect and, as a result, is not willing to corner the upstream market. There exists a monopoly-like equilibrium when downstream products are sufficient substitutes. By the reverse token, only the perfect competition outcome emerges when the competition on the downstream market is sufficiently weak. In other words, there is a tension between competitiveness on the downstream market and competitiveness on the upstream market. Intuitively, the same downstream interactions which strengthen the competitive pressure on the downstream market, are those which soften the competitive pressure on the upstream market.

This tension is revealed in downstream prices, which turn out to be non-monotonic in the substitutability parameter (provided that a monopoly-like equilibrium is selected when it exists). The level of downstream prices results indeed from two combined forces: the level of upstream prices on the one hand, and the intensity of downstream competition / substitutability on the other hand.

3.3 Partial Foreclosure and Entrant's Efficiency

In this section, we show that the efficiency of the downstream entrant is another important ingredient for the emergence of partial foreclosure equilibria. To make this point, we consider the Hotelling-Salop case. We assume that the three firms are localized symmetrically on the Salop (1979) circle.[23] Transport costs, parameterized by t, are linear, and there is a mass one of consumers uniformly located on the circle. We assume that the utility derived from consuming the downstream product is sufficiently high relative to the transport cost, so that the final market is always covered. Both integrated firms have the same linear cost function $c_i(q) = c_j(q) = cq$, while downstream firm d's cost is given by $c_d(q) = (c + \delta)q$, where δ may be positive or negative.

In the following, we assume that the price offered by the alternative source of input is not too low, in a sense that is made more precise in Appendix A.8.[24] We obtain the following proposition:

Proposition 4. *Consider the Hotelling-Salop case:*

- *If* $\delta \geq -\frac{t}{9}$, *there exist monopoly-like equilibria.*
- *Otherwise, there are no partial foreclosure equilibria.*

Proof. See Appendix A.8. □

[23] We would obtain similar results if we used the Shubik and Levitan (1980) demands, as in the previous section. However, we would have to resort to numerical simulations.

[24] Broadly speaking, there exist situations, in which, for a given c_d, when $\overline{m}$ is quite low, no partial foreclosure equilibria exist, whereas they exist when $\overline{m}$ is high enough. We come back to this point when we discuss the impact of a price cap in Section 4.

Put differently, a less efficient downstream firm is more likely to be partially foreclosed. Everything else equal, when firm d operates with a high marginal cost, it serves a relatively small fraction of final consumers. This implies that the profits earned from input sales are low, which reduces the incentives to undercut the upstream market. On the other hand, the softening effect is not directly affected by the entrant's efficiency, as it depends merely on how much firm d's demand increases when the upstream supplier raises its downstream price. As a result, an increase in the entrant's marginal cost lowers the incentives to undercut, and increases the scope for monopoly-like equilibria.

3.4 Discussions and Extensions

Two-part tariffs In the following, we claim that partial foreclosure equilibria with positive upstream profits still exist when firms are allowed to use two-part tariffs on the upstream market. For $i = 1, 2$, denote by w_i and F_i the variable and fixed parts of the tariff respectively. To simplify the exposition, we make the following assumptions:

- The downstream firm is not able to switch to another supplier at the end of stage 2. If we allowed firm d to do so, then, its optimal choice between tariffs (w_i, F_i) and (w_j, F_j) at the end of stage 2 would depend on the downstream prices that have just been set.[25] These downstream prices would depend in turn on the anticipated choice of upstream supplier. Forbidding firm d to switch allows us to avoid these (unnecessary) complications.

- The alternative source of input allows the downstream firm to obtain positive, but arbitrarily small profits: $\pi_d^{(\emptyset)}(\overline{m}) = \varepsilon > 0$.

As in Bonanno and Vickers (1988), let us define w_{tp}, the upstream price that maximizes the upstream supplier and the downstream firm's joint profits: $w_{tp} \equiv \arg\max_w \pi_i^{(i)}(w) + \pi_d^{(i)}(w)$. It is easy to adapt the proof of Lemma 3, to show that $w_{tp} > m$. Let us first solve for the equilibria without putting any restrictions on the fixed part of the tariff.

Clearly, in any equilibrium, the variable part of the upstream supplier's tariff has to be equal to w_{tp}. Otherwise, the upstream supplier could profitably deviate by setting w_{tp} and adjusting the fixed fee, by definition of w_{tp}. Consider first that $\pi_i^{(i)}(w_{tp}) + \pi_d^{(i)}(w_{tp}) \leq \pi_j^{(i)}(w_{tp})$. Then, there exists an equilibrium, in which firm i offers the tariff $(w_{tp}, \pi_d^{(i)}(w_{tp}))$,[26], where the fixed fee fully extracts firm d's profit, while integrated firm j chooses to make no upstream offer: this is similar to a monopoly-like equilibrium.[27] In this case, upstream profits are obviously positive.

On the other hand, if $\pi_i^{(i)}(w_{tp}) + \pi_d^{(i)}(w_{tp}) > \pi_j^{(i)}(w_{tp})$, then, the above equilibrium can no longer be sustained, since firm j would rather undercut. In this case, there exists an equilibrium, in which both integrated firms set a variable part equal to w_{tp}, and a fixed fee equal to $\pi_j^{(i)}(w_{tp}) - \pi_i^{(i)}(w_{tp})$,

[25] Whereas, under linear tariff competition, the downstream firm always goes for the cheapest offer.

[26] This is where our assumption that $\pi_d^{(\emptyset)}(\overline{m})$ can be made arbitrarily small comes in. Without this assumption, we would have to take into account the participation constraint of firm d.

[27] Other equilibria exist. For instance: $w_j = m$, $F_j = F$, where F is neither too large nor too low; and $w_i = w_{tp}$, $F_i = \pi_d^{(i)}(w_{tp}) - \pi_d^{(i)}(m) + F$. These equilibria seem quite fragile. In particular, they vanish if we assume that firm d 'trembles' when it chooses its upstream supplier.

so that both firms earn $\pi_j^{(i)}(w_{tp})$.[28] In this case, upstream profits are positive as well. Indeed, they can be written as:

$$
\begin{aligned}
[w_{tp} - m]\, q_d(p^{(i)}(w_{tp})) + \pi_j^{(i)}(w_{tp}) - \pi_i^{(i)}(w_{tp}) \quad = \quad & \left[p_j^{(i)}(w_{tp}) - m\right] q_j(p^{(i)}(w_{tp})) - c_j\left(q_j(p^{(i)}(w_{tp}))\right) \\
& - \left[p_i^{(i)}(w_{tp}) - m\right] q_i(p^{(i)}(w_{tp})) + c_i\left(q_i(p^{(i)}(w_{tp}))\right),
\end{aligned}
$$

which is strictly positive by Proposition 1.

These results are summarized in the following proposition:

Proposition 5. *Under two-part tariff competition, partial foreclosure arises in any equilibrium. Besides, there exist equilibria with positive upstream profits.*

Proof. Immediate. □

Consider now that negative fixed fees are not feasible. Then, the outcomes described above remain equilibria, as long as $\pi_i^{(i)}(w_{tp}) \leq \pi_j^{(i)}(w_{tp})$, namely, as long as the softening effect is strong enough. If, on the other hand, this inequality is not satisfied, then, these equilibria are no longer sustainable.

Downstream strategic interactions. In line with the vertical mergers literature, we have assumed so far that downstream prices are strategic complements. This is however not a crucial assumption. On the contrary, we argue that strategic substitute prices would strengthen the softening effect and thus increase the scope for partial foreclosure. Let us informally explain why, by considering the downstream competition stage. The upstream supplier has incentives to raise its downstream price to preserve its upstream profit. When prices are strategic complements, the integrated rival best responds by raising its downstream price as well, which reduces the gap between equilibrium downstream prices and weakens the softening effect. By contrast, when prices are strategic substitutes, the integrated rival lowers its downstream price, which enlarges the gap between equilibrium downstream prices and strengthens the softening effect.

Quantity competition. The softening effect exists if the upstream supplier can enhance its upstream profits by behaving softly on the downstream market. As discussed previously, this requires that it actually interacts with the unintegrated downstream firm. One may wonder whether the softening effect hinges on the assumption of price competition on the downstream market, for if the downstream strategic variables are quantities and all firms play simultaneously, then the upstream supplier can no longer impact its upstream profit through its downstream behavior. However, if for instance integrated firms are Stackelberg leaders on the downstream market, then the upstream supplier's quantity choice modifies its upstream profit, and the softening effect is still at work. To summarize, the question is not whether firms compete in prices or in quantities, but whether the strategic choice of a firm can affect its rivals' quantities.[29]

[28] Other equilibria similar to the ones described in footnote 27 also exist.

[29] With a linear demand function and quantity competition, if integrated firms are Stackelberg leaders on the downstream market, then a monopoly-like equilibrium always exists.

4 Welfare and Regulation

As the following proposition shows, partial foreclosure equilibria can significantly degrade consumers' surplus and social welfare:

Proposition 6. *Consumers strictly prefer the perfect competition outcome to a partial foreclosure equilibrium.*
Besides, if firms' downstream divisions are identical[30] *and downstream costs are weakly convex, then, social welfare is strictly higher in the perfect competition outcome than in a partial foreclosure equilibrium.*

Proof. See Appendix A.9. □

Strategic complementarity ensures that all prices increase when the industry shifts from the perfect competition outcome to a partial foreclosure equilibrium. In this case, a partial foreclosure equilibrium is clearly detrimental to all consumers.

Assessing the impact on social welfare requires more assumptions. If the assumptions made in Proposition 6 are satisfied, then, a shift from the perfect competition outcome to a partial foreclosure equilibrium has the following implications. First, since all prices go up due to strategic complementarity, the total quantity produced diminishes. This is clearly welfare-degrading, since the total demand is already too low at the perfect competition outcome, due to positive markups on the downstream market. Second, the outcome on the final market becomes more asymmetric: firms i and d have more incentives to increase their downstream prices than firm j. This merely shifts some demand from firms i and d to firm j, which is again detrimental to welfare if downstream costs and preferences are convex.

Since partial foreclosure equilibria can degrade both social welfare and consumers' surplus, there is a rationale for regulatory intervention. In several countries (e.g., France, Spain, Belgium, Italy), the telecoms regulator sets a price at which the broadband incumbent has to supply any service-based firm. This does not prevent the incumbent from negotiating lower tariffs with downstream firms. Therefore, the regulated price can be seen as a price cap on the incumbent's wholesale offer. In the following, we show that this kind of regulation can favor the development of tough wholesale competition, and remove all partial foreclosure equilibria, even if the price cap is strictly above marginal cost.

As a first step, let us inspect Figure 2, which depicts firms' profits in the symmetric linear case. Notice that for any $w_i \in (m, w_*)$, $\pi_i^{(i)}(w_i) > \pi_j^{(i)}(w_i)$: in this range of upstream prices, it is always better to be the upstream supplier. Consequently, if the regulator sets any price cap between m and w_*, then, the only equilibrium is the perfect competition outcome.

Now we would like to extend this result to more general demand and cost systems. To do so, we have to compare $\pi_i^{(i)}(w)$ and $\pi_j^{(i)}(w)$ for w_i slightly above m. Put differently, we need to derive conditions under which $\frac{d\pi_i^{(i)}}{dw}(m) > \frac{d\pi_j^{(i)}}{dw}(m)$. We obtain the following proposition:

[30]Namely, if downstream demands are symmetric, and cost functions are the same for the three firms.

Proposition 7. *Assume that firms' downstream divisions are identical and downstream costs are weakly convex. Then, a low enough price cap, strictly above the upstream marginal cost, destroys all partial foreclosure equilibria if*

- $\frac{\partial^2 q_k}{\partial p_k^2} \leq 0$ *for all* $k \in \{1, 2, d\}$.
- *or,* $\frac{\partial^2 q_k}{\partial p_k \partial p_{k'}} \geq 0$ *for all* $k \neq k'$ *in* $\{1, 2, d\}$.

Proof. See Appendix A.10. □

A price cap strictly larger than the upstream marginal cost can restore the competitiveness of the wholesale market, provided that the upstream supplier earns more profits than its integrated rival when the upstream price is slightly above the marginal cost. Put differently, the upstream profit effect has to dominate the softening effect for w_i sufficiently close to c_u. A good proxy to assess the strength of the softening effect is the difference between the upstream supplier's and the integrated rival's downstream prices. This gap is small if the upstream supplier does not raise its downstream price by much when the upstream price increases, which is the case when a firm's demand is concave with respect to its own price, and downstream costs are convex. Besides, given strategic complementarity, the integrated rival increases its price as well, which implies an even smaller gap between downstream prices, hence, a small softening effect. This is the first sufficient condition in Proposition 7.

Second, even if the upstream supplier does increase its price a lot, the gap may still be small if the integrated rival reacts by also increasing its price a lot, namely, if downstream prices are strongly strategic complements. A sufficient condition for this is $\partial^2 q_k / \partial p_k \partial p_k' \geq 0$ and convex costs. This is the second condition in the proposition.[31]

We would like to emphasize that Proposition 7 does not come from a simple mechanical effect. Of course, imposing a price cap reduces the upstream price mechanically. But, more fundamentally, under the assumptions detailed in Proposition 7, a price cap initiates a process by which integrated firms will undercut each other, leading to tough competition in the wholesale market. Interestingly, a price cap can influence the outcome of the market even though the regulatory constraint does not bind (i.e., the upstream price is strictly smaller than the price cap) in equilibrium. Note also that it is sufficient to impose a price cap on one of the integrated firms only to fuel competition in the wholesale market.

Notice that the threat of investment by firm d can have the same impact as a price cap on the wholesale market. Consider the following alteration of our game: between stage 1 and stage 2, after having observed the integrated firms' upstream offers, the unintegrated downstream firm can pay a sunk investment cost to build its own network. If it does so, it becomes able to produce the intermediate input at marginal cost m. If the investment cost is not too large, there is a threshold

[31] It should be noticed that this reasoning, which derives conditions for the upstream profit effect to dominate the softening effect, is only valid in the neighborhood of m. Therefore, the sufficient conditions given in Proposition 7 do not imply that partial foreclosure equilibria do not exist. For instance, in the symmetric linear case, both sufficient conditions hold and monopoly-like equilibria exist when γ is high enough.

$\bar{w}$, such that firm d invests if, and only if the cheapest wholesale offer is above $\bar{w}$. Since integrated firms prefer to face a relatively less efficient competitor, at least one integrated firm will make an offer below $\bar{w}$ to prevent firm d from investing: firms behave exactly as if $\bar{w}$ were a price cap. If the cost of bypass is low, then $\bar{w}$ is low as well, and, under the assumptions of Proposition 7, only the perfect competition outcome emerges.

This result has interesting policy implications. In the mobile industry, it means that favorable terms for spectrum licences (e.g., terms for ungranted mobile licences, or for Wimax licences) can increase MNOs' incentives to set low wholesale prices for MVNOs. In the broadband market, it implies that favorable conditions for local loop unbundling investments (e.g., low rates for colocation in the historical operator's premises) might stimulate the development of the wholesale broadband market.

5 Market Structure and Partial Foreclosure

We now study the impact of the market structure on the emergence of partial foreclosure equilibria. This section serves two purposes. First, it shows that the results derived before are robust, in the sense that partial foreclosure equilibria can still exist with more integrated or downstream firms. Second, it derives some interesting comparative statics on the impact of the market structure on the likelihood of partial foreclosure.

We assume that there are $M \geq 2$ integrated firms, denoted by $1, 2, \ldots M$, and $N \geq 1$ downstream firms, denoted by $d_1, d_2, \ldots d_N$. This generalization introduces several complications. To begin with, if we allowed integrated firms to make discriminatory offers on the input market, we would have to keep track of the corresponding $M \times N$ upstream prices. The number of potential equilibria, and the number of potential deviations to check would then be quite large, which would make the model much harder to solve. To get around this issue, we assume in the following that integrated firms can only make non-discriminatory offers on the upstream market. As before, we denote by w_i the upstream price set by firm $i \in \{1, 2, \ldots M\}$. In line with the assumptions we made before, we assume that the alternative source of input allows all downstream firms to be active on the final market.

The second complication comes from the fact that, contrary to the basic framework with only one downstream firm, when, several integrated firms offer the same upstream price w, where $w = \min_{i \in \{1,2,\ldots M\}}$, these upstream offers are no longer equivalent. To see this, consider, for instance, that there are two integrated firms, 1 and 2, which both charge $w > m$, and two unintegrated downstream firms, d_1 and d_2. Assume that firm d_1 purchases the input from firm 1. The intuition underlying Proposition 1 is still present, so that firm 1, being firm d_1's upstream supplier, behaves less aggressively on the downstream market. Now, consider d_2's choice of upstream supplier. Firm d_2 can either purchase from 1 to make it an even softer downstream competitor, or it can buy from 2 to make it a soft competitor as well. It is unclear which strategy is optimal, i.e., whether the choices of upstream supplier are strategic complements or strategic substitutes, but in general, d_2's optimal choice depends on d_1's upstream supplier. Put differently, the choices of upstream supplier are part of a strategic game between the unintegrated downstream firms.

As it turns out, when demand functions are linear, a downstream firm's optimal choice of supplier does not depend on the others' choices.[32] We will therefore focus on that case for the sake of simplicity. This implies that any repartition of the input demand among the cheapest upstream suppliers is supported by equilibrium strategies. Another consequence of this simplification is to generate a lot of potential equilibria with every possible asymmetric outcome on the upstream market.[33] Since a complete characterization of all the equilibria requires cumbersome notations with no conceptual difficulty or meaningful economic interpretation, we focus here on two polar types of partial foreclosure equilibria.

In the first type of (potential) foreclosure equilibria, the upstream market is supplied by only one integrated firm, i, at the monopoly upstream price,[34] while the other integrated firms make no upstream offer. This is a monopoly-like outcome. To investigate whether deviations are profitable, we have to specify what happens when one of the other integrated firms matches the upstream offer of the input supplier. In this case, we assume that downstream firms coordinate on an equilibrium in which they all purchase from firm i. As pointed out before, this outcome is indeed an equilibrium when demands are linear, since downstream firms are then indifferent between the two offers. The monopoly-like outcome can be sustained in equilibrium if, and only if, the integrated firms which do not supply the upstream market earn more total profits than the upstream supplier.[35] As in Section 3, this condition may well be satisfied, since these other integrated firms earn higher downstream profits than the upstream supplier, due to the softening effect. There is a monopoly-like equilibrium if the softening effect outweighs the upstream profit effect.

The other polar case of foreclosure equilibria is as follows. All integrated firms offer the same upstream price $w > m$ and each of them supplies a fraction $(N - M)/M$ of the input demand, ignoring integer constraints. We refer to these situations as *collusive-like outcomes*. When an integrated firm deviates upward, we assume that the input demand is split equally among the $M - 1$ other integrated firms. Again, this new repartition of the upstream demand is part of a subgame-perfect equilibrium when demands are linear. The proposed collusive-like outcome is part of an equilibrium if, and only if, no integrated firm is willing to undercut the upstream market, nor to take back its upstream offer. The former condition is met if an integrated firm's benefits from the soft behavior of its integrated rivals on the downstream market outweigh the additional upstream profits from undercutting. The latter condition is met when the upstream profit is large enough to deter an integrated firm to remove its offer.[36]

[32] This statement is made more precise in Lemma 8, and proven in Appendix A.11.

[33] Firm 1 supplies all downstream firms; or firms 1 and 2 share the upstream demand equally; or firms 1, 2 and 3 supply 1/2, 1/4 and 1/4 of the upstream demand respectively; or ...

[34] The definition of this price follows readily from the definition of w_m in Section 2.

[35] Formally, denoting by $\pi_i^{(i)}(w)$ and $\pi_j^{(i)}(w)$ the profits of the upstream supplier and of the other integrated firms, respectively, when the upstream price is w, there is a monopoly-like equilibrium if, and only if, $\pi_j^{(i)}(w_m) \geq \pi_i^{(i)}(w_m)$, where $w_m = \arg\max_{w \leq \overline{m}} \pi_i^{(i)}(w)$.

[36] Formally, denoting by $\pi_1^{coll}(w)$ the profit of an integrated firm when the upstream market is equally shared among all integrated firms at price w, and by $\pi_2^{coll}(w)$ the profit of an integrated firm when the upstream market is equally shared between all the *other* integrated firms at price w, there is a collusive-like equilibrium with an upstream price

These outcomes are called collusive-like, because, to an outside observer, they look like collusion. All the upstream competitors stick to the same upstream price, and this price is strictly higher than what a standard single market analysis would predict. Moreover, the upstream profits are equally shared between the integrated firms. Yet, this poorly competitive outcome is sustained without any agreements or repeated interactions between vertically integrated firms. They simply do not to undercut their integrated rivals, because they benefit from their soft behaviors on the downstream market.

As already mentioned, we derive our results under a linear specification of the demand functions. More precisely, demands are derived from the following model of spatial competition. Each firm is linked to each of its $M+N-1$ rivals by a segment of length $\frac{2}{(M+N)(M+N-1)}$. A mass 1 of consumers is uniformly located on these $\frac{(M+N)(M+N-1)}{2}$ segments. Each consumer purchases zero or one unit of the downstream product. Transport costs, parameterized by t, are linear, and we assume that the utility derived from consumption of the downstream good is sufficiently high, so that the market is always fully covered.

Notice that, when there are only two firms competing on the downstream market, this model is equivalent to the Hotelling segment. Similarly, when only three firms are present, we obtain the Salop (1979) circle model. With more firms, this equivalence no longer holds, since, in our model, each firm competes with all its rivals, whereas in the Salop circle model, each firm only competes with its two neighbors. One of the advantages of our model of non-localized competition over the Salop model is that we do not need to choose the locations of our $M+N$ firms. If we used instead the Salop model, our results would depend on the way downstream firms are located with respect to integrated firms.

When a new firm is added to the downstream market, we assume that $M+N$ new segments are created, and that some consumers are relocated on these new segments, so that there is still a mass 1 of consumers uniformly localized. This assumption may seem odd, but it is similar in spirit to Salop (1979)'s assumption that firms relocate symmetrically on the circle following entry. The alternative would be to assume that new consumers, which did not consume previously are added to the new segments, so that the downstream demand would grow unboundedly with the number of firms.

Solving for the locations of marginal consumers, we deduce the demand addressed to each firm:

$$q_k = \frac{1}{M+N} + \frac{1}{2t}\sum_{k' \neq k}(p_{k'} - p_k), \tag{6}$$

with $k \in \{1, 2, \ldots, M, d_1, \ldots, d_N\}$. We assume that all integrated firms have the same downstream marginal cost c, whereas unintegrated downstream firms operate with marginal cost $c+\delta$, where δ can be positive or negative. We already know from Proposition 4 that, when $M=2$ and $N=1$,

w if, and only if, $\pi_1^{coll}(w) \geq \max\{\sup_{\tilde{w}<w} \pi_i^{(i)}(\tilde{w}), \pi_2^{coll}(\tilde{w})\}$.

Notice that, by continuity, when an upstream price w satisfies these two inequalities strictly, all the upstream prices in the neighborhood of w also do so. Therefore, in non degenerated situations, there is a continuum of collusive-like equilibria.

there is a threshold $\overline{\delta}$ above which partial foreclosure outcomes can arise in equilibrium. In the following proposition, we show that this result extends to market structures with $M > 2$ or $N > 1$:

Proposition 8. *Consider the demand functions* (6) *with cost parameter* δ. *For all* $M \geq 2$ *and* $N \geq 1$, *there exists a threshold* $\delta^m(M, N)$, *such that monopoly-like equilibria exist if, and only if,* $\delta \geq \delta^m(M, N)$.
There also exist two thresholds, $\underline{\delta}^{coll}(M, N) < \overline{\delta}^{coll}(M, N)$, *such that collusive-like equilibria exist if, and only if,* $\underline{\delta}^{coll}(M, N) \leq \delta \leq \overline{\delta}^{coll}(M, N)$.
These thresholds are ranked as follows:

$$\underline{\delta}^{coll}(M, N) < \delta^m(M, N) < \overline{\delta}^{coll}(M, N).$$

Proof. See Appendix A.11 for the threshold $\delta^m(M, N)$. The proof for thresholds $\underline{\delta}^{coll}(M, N)$ and $\overline{\delta}^{coll}(M, N)$ is lengthy and tedious. A Mathematica file including all the computations is available online at `http://sites.google.com/site/nicolasschutz/jmp` □

According to Proposition 8, there is always a range of cost parameters such that monopoly-like or collusive-like equilibria exist. In other words, the results derived in Section 3 are not specific to the case with two integrated firms and one downstream firm. Whatever the number of integrated and downstream firms, the decision to undercut the upstream market trades off the softening effect against the upstream profit effect, and the perfect competition outcome does not necessarily emerge. When the softening effect is strong enough, which happens when $\delta \geq \delta^m(M, N)$, the incentives to undercut are weak, and monopoly-like equilibria exist.

A similar insight holds for collusive-like equilibria. When $\delta \geq \underline{\delta}^{coll}(M, N)$, it is not profitable to undercut a collusive-like outcome. However, the softening effect should not be too strong: when $\delta \geq \overline{\delta}^{coll}(M, N)$, the softening effect is so strong that an integrated firm prefers not to make any upstream offer, rather than taking part in a collusive-like equilibrium. As pointed out before, these equilibria look like collusion. For instance, when $M = 2$, $N = 2$ and $\delta = 0$, there exists an equilibrium, in which firm 1 supplies firm d_1 and firm 2 supplies firm d_2 at a price strictly above marginal cost. Integrated firms do not want to undercut, since they do not want to lose the softening effect. They do not want to exit the upstream market either, since they also want to enjoy some upstream profits. These equilibria can also be interpreted in terms of second sourcing. When $M = 2$, $N = 1$ and $\delta = 0$, there is an equilibrium in which downstream firm d_1 purchases the input above marginal cost from both integrated firms.

To summarize, when $\delta < \underline{\delta}^{coll}(M, N)$, there are neither monopoly-like nor collusive-like equilibria. When $\underline{\delta}^{coll}(M, N) \leq \delta < \delta^m(M, N)$, only collusive-like equilibria exist. When $\delta^m(M, N) \leq \delta \leq \overline{\delta}^{coll}(M, N)$, both collusive-like and monopoly-like equilibria exist. Last, when $\delta > \overline{\delta}^{coll}(M, N)$, only monopoly-like equilibria exist.

We now show that an increase in the number of firms, integrated or not, can actually make partial foreclosure equilibria more likely. We perform several types of comparative statics on monopoly-like and collusive-like equilibria. We first analyze the impact of M and N on the cost thresholds. Then, we investigate the consequences of the vertical integration of an unintegrated downstream firm. To do so, we denote by $L = M + N$ the total number of firms, and we analyze the behavior of $\delta^m(M, L-M)$ and $\underline{\delta}^{coll}(M, L-M)$ as a function of M.[37] Notice that, for this particular comparative statics, our results do not depend on how the total demand is affected by an increase in the number of firms. We prove the following proposition:

Proposition 9. *Thresholds $\delta^m(.,.)$ and $\underline{\delta}^{coll}(.,.)$ evolve as follows:*

- Number of integrated firms:
  $M \mapsto \delta^m(M, N)$ *and* $M \mapsto \underline{\delta}^{coll}(M, N)$ *are hump-shaped.*

- Mix between integrated and downstream firms:
  $M \mapsto \delta^m(M, L - M)$ *is increasing for* $L = 4$ *and hump-shaped otherwise.*
  $M \mapsto \underline{\delta}^{coll}(M, L - M)$ *is increasing for* $L \leq 6$ *and hump-shaped otherwise.*

- Number of downstream firms:
  $N \mapsto \delta^m(M, N)$ *is hump-shaped for* $M = 2$*, and decreasing otherwise.*
  $N \mapsto \underline{\delta}^{coll}(M, N)$ *is increasing for* $M = 2$*, and decreasing otherwise.*

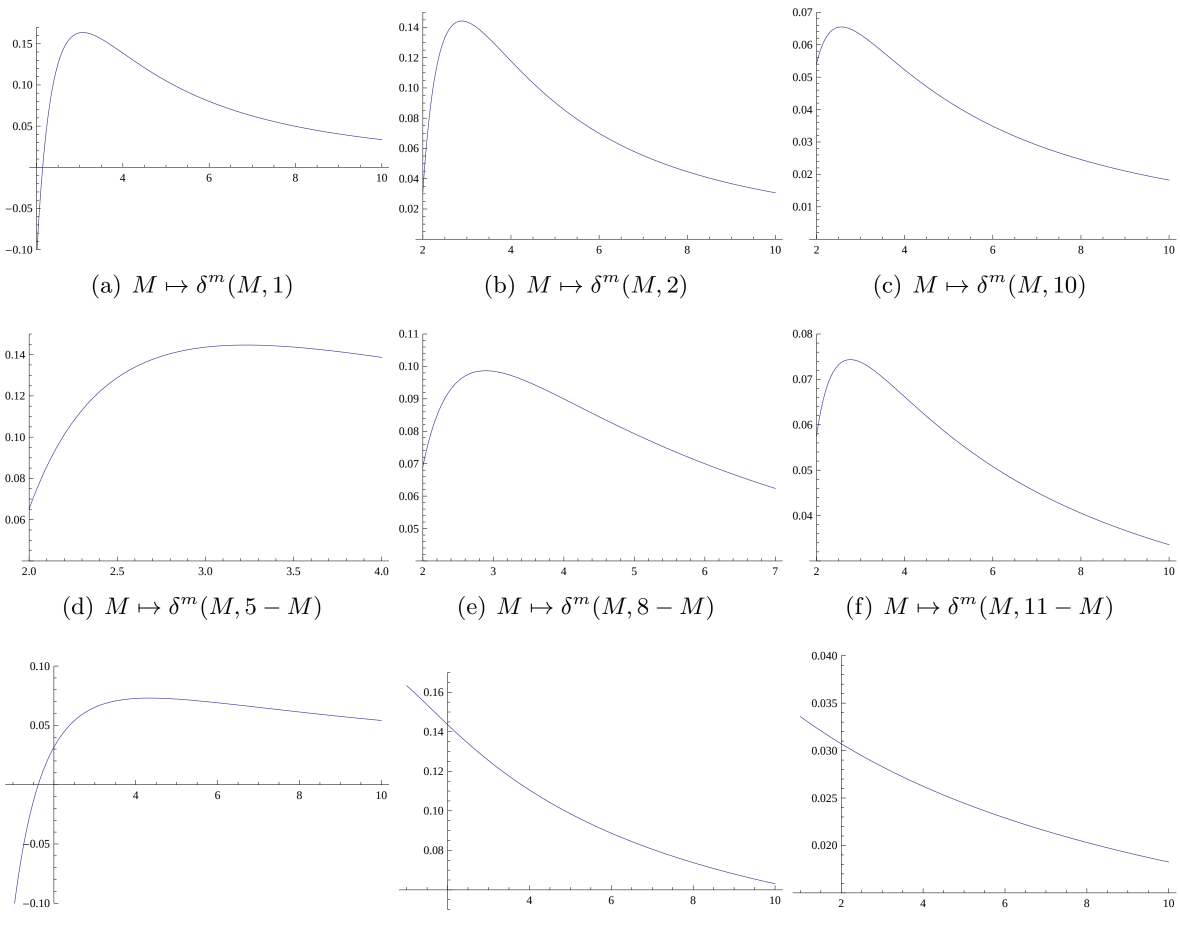
Proof. See Appendix A.12. Again, the results for threshold $\underline{\delta}^{coll}$ are proven in a Mathematica file. □

In Figures 3 and 4, we plot the thresholds δ^m and $\underline{\delta}^{coll}$ for several values of the parameters.

An increase in the number of integrated firms from 2 to 3 tends to have a strong positive impact on the monopoly-like and collusive-like thresholds. However, further increases in M imply a mild decrease in these thresholds. Put differently, the impact of the number of integrated firms on the emergence of partial foreclosure equilibria is non-monotonic (panels (a), (b) and (c) on Figures 3 and 4). Intuitively, as M increases, the softening effect gets weaker: when the upstream supplier raises its downstream price, the proportion of consumers that switch to the downstream firms is lower when more firms are competing in the market. But the upstream profits decrease as well, as downstream firms suffer more from downstream competition. With demand functions (6), the former effect dominates when M is initially low, whereas the latter dominates for higher values of M. This reasoning also applies when the total number of firms is fixed, which is the reason why $M \mapsto \delta^m(M, L - M)$ and $M \mapsto \underline{\delta}^{coll}(M, L - M)$ are both hump-shaped (panels (d), (e) and (f) on Figures 3 and 4).

An increase in the number of downstream firms has a negative impact on cost thresholds δ^m and $\underline{\delta}^{coll}$ as long as $M \geq 3$ (panels (g), (h) and (i) on Figures 3 and 4). This result bears some similarities with Proposition 3, which highlighted the tension between upstream and downstream competitiveness. More downstream firms strengthens the softening effect, as an increase in the upstream suppliers' downstream price translates into a higher increase in the input demand. An

[37] We are not interested in the behavior of $\overline{\delta}^{coll}(M, L - M)$, since we know from Proposition 8 that this threshold is strictly larger than the monopoly-like cost threshold. This means that, when δ is so high that collusive-like equilibria fail to exist, there are always monopoly-like equilibria.



(a) $M \mapsto \delta^m(M,1)$ (b) $M \mapsto \delta^m(M,2)$ (c) $M \mapsto \delta^m(M,10)$

(d) $M \mapsto \delta^m(M,5-M)$ (e) $M \mapsto \delta^m(M,8-M)$ (f) $M \mapsto \delta^m(M,11-M)$

(g) $N \mapsto \delta^m(2,N)$ (h) $N \mapsto \delta^m(3,N)$ (i) $N \mapsto \delta^m(10,N)$

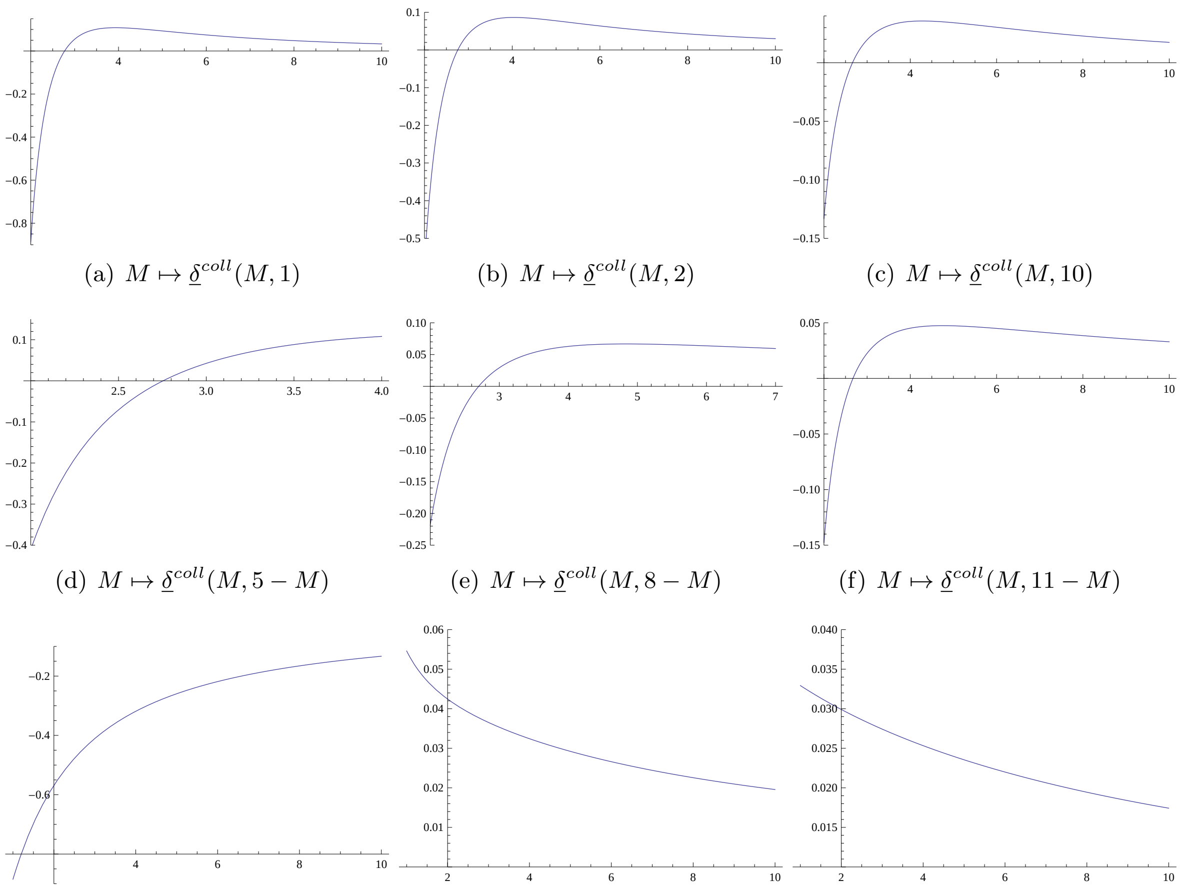
Figure 3: Monopoly-like equilibria

increase in N also raises the input demand, and hence, the upstream profits. The increase in the softening effect tends to dominate when $M \geq 3$, whereas the increase in upstream profits sometimes dominate when $M = 2$.

These results point to the following conclusion. There is no reason to expect an increase in the number of integrated firms, or an increase in the number of downstream firms, or a change in the mix between downstream firms and integrated firms, to always make partial foreclosure less likely. When the market structure changes, the softening effect and the upstream profit tend to vary in the same direction, thereby leading to ambiguous predictions for the emergence of partial foreclosure.

6 Complete Foreclosure

In this section, we relax the alternative source of input assumption, and we investigate whether there can exist equilibria, in which the downstream firm does not receive any upstream offer. Put differently, we wonder whether situations in which downstream firm d is completely foreclosed can arise in equilibrium. Before deriving our results with two integrated firms and one downstream



(a) $M \mapsto \underline{\delta}^{coll}(M,1)$ (b) $M \mapsto \underline{\delta}^{coll}(M,2)$ (c) $M \mapsto \underline{\delta}^{coll}(M,10)$

(d) $M \mapsto \underline{\delta}^{coll}(M,5-M)$ (e) $M \mapsto \underline{\delta}^{coll}(M,8-M)$ (f) $M \mapsto \underline{\delta}^{coll}(M,11-M)$

(g) $N \mapsto \underline{\delta}^{coll}(2,N)$ (h) $N \mapsto \underline{\delta}^{coll}(3,N)$ (i) $N \mapsto \underline{\delta}^{coll}(10,N)$

Figure 4: Collusive-like equilibria

firm, we analyze a similar issue in the upstream bottleneck benchmark, which we present in the following.

6.1 Upstream Bottleneck Benchmark

In this subsection, we assume that there is only one vertically integrated firm, denoted by 1, and an unintegrated downstream firm, denoted by d. For simplicity, we assume that downstream costs are linear, and we denote by c_k the downstream marginal cost of firm $k \in \{1, d\}$. The timing is the same as before. In stage 1, firm 1 announces the input price at which it is ready to supply firm d. In stage 2, firms compete in prices. We make the same assumptions as in Section 2 about the demand and profit functions: a firm's demand is decreasing with respect to its price and increasing with respect to its rival's price; firms are symmetric; demands have a finite choke point; the total demand is non-increasing with respect to prices; firms' best responses are unique; for each w, there exists a unique Nash equilibrium for the downstream competition subgame; downstream prices are strategic complements.

As before, we say that firm 1 offers $+\infty$, or that it makes no upstream offer, when it proposes a

w that does not allow firm d to compete on the final market. The profit functions at the downstream competition stage are denoted by:

$$\begin{aligned}\tilde{\pi}_1^{(1)}(p_1, p_d, w) &\equiv (p_1 - c_1 - m)q_1(p_1, p_d) + (w - m)q_d(p_1, p_d),\\ \tilde{\pi}_d^{(1)}(w)(p_1, p_d, w) &\equiv (p_d - c_d - w)q_d(p_1, p_d),\end{aligned}$$

when firm 1 supplies the input at price w. When firm 1 makes no upstream offer, its profit is given by $\tilde{\pi}_1^{(\emptyset)}(p_1) \equiv (p_1 - c_1 - m)q_1(p_1, +\infty)$, and firm d earns $\tilde{\pi}_d^{(\emptyset)} = 0$. Denoting by $p_k^{(1)}(w)$, $k = 1, d$, the downstream equilibrium prices, the profits at the downstream equilibrium are given by:

$$\pi_k^{(1)}(w) \equiv \pi_k(p_1^{(1)}(w), p_d^{(1)}(w), w).$$

When firm 1 does not supply the input to firm d, it solves the standard monopoly problem: $\max_{p_1} \tilde{\pi}_1^{(\emptyset)}(p_1)$. We denote by p^m the monopoly downstream price of firm 1, and we assume that p^m is unique.

When firm 1 considers whether to supply the input to firm d, it simply compares $\tilde{\pi}_1^{(\emptyset)}(p^m)$ and $\arg\max_w \pi_1^{(1)}(w)$. If the former is larger than the latter, then, complete foreclosure arises in equilibrium. The following proposition shows that this outcome is unlikely in the upstream bottleneck benchmark:

Proposition 10. *Consider the upstream bottleneck benchmark. If*

$$q_d(p^m, p^m + c_d - c_1) > 0, \tag{7}$$

then, complete foreclosure does not arise in equilibrium.

Proof. See Appendix A.13. □

When firm 1 considers whether to supply the downstream firm, it trades off two effects: the cannibalization effect, and the softening effect. Proposition 10 tells us that, when condition (7) is satisfied, the integrated firm can control the strength of the cannibalization effect by setting a high input price, while still earning some positive upstream profits, so that complete foreclosure does not arise. Intuitively, if the integrated firm offers an input price equal to $w = p^m - c_1 + \varepsilon$ with $\varepsilon > 0$, and sets a downstream price equal to p^m, it earns profits

$$(p_m - c_1 - m)\left(q_1(p^m, p_d) + q_d(p^m, p_d)\right) + \varepsilon q_d(p^m, p_d). \tag{8}$$

Under condition (7), and taking ε as close to zero as needed, there always exists a $p_d > w + c_d$ such that $q_d(p^m, p_d) > 0$, so that the entrant can earn positive profits. In this case, the profit in expression (8) is strictly larger than firm 1's downstream monopoly profit, and complete foreclosure is not profitable.[38]

[38]The actual proof is slightly more complicated, since we have to ensure that downstream prices are Nash equilibrium

Notice that, when $c_1 = c_d$, $q_d(p^m, p^m + c_d - c_1) = q_d(p^m, p^m)$, and therefore, condition (7) clearly holds. Put differently, when c_d is not too high relative to c_1, the no-foreclosure strategy outlined above is feasible. This implies that complete foreclosure never arises, as long as downstream firm d is not too inefficient. In the following, we will see that this result may not hold anymore when two integrated firms are present. In other words, having more integrated firms may make complete foreclosure more likely.

6.2 Complete Foreclosure and Upstream Competition

Now, let us go back to our initial market structure, with two integrated firms, one unintegrated downstream firm and no alternative source of input. For simplicity, let us assume that downstream costs are linear. We still assume that the duopoly demand functions are symmetric, namely, when firm d is not active, $q_1^D(p_1, p_2) = q_2^D(p_2, p_1)$. However, we will allow for some asymmetry between the integrated firms when $q_d > 0$. This will enable us to derive some interesting comparative statics results on the impact of input differentiation on the emergence of complete foreclosure. Apart from that, all the assumptions we made in Section 2 are satisfied.

Since there is no alternative source of input, the payoff functions of firms 1 and 2, when the downstream firm does not manage to obtain the input, are given by $\tilde{\pi}_i^{(\emptyset)}(p_1, p_2) \equiv (p_k - c_1 - m)q_k^D(p_1, p_2)$, $i = 1, 2$, while the pure downstream firm earns $\tilde{\pi}_d = \pi_d = 0$. As before, we denote by $p_i^{(\emptyset)}$ the downstream equilibrium prices, while $\pi_i^{(\emptyset)}$ is the equilibrium profit of firm i. With these notations, there exists a subgame-perfect equilibrium with complete foreclosure, if, and only if, for all w, for all $i = 1, 2$, $\pi_i^{(i)}(w) \leq \pi_i^{(\emptyset)}$.

Now, it is interesting to understand how the trade-off between foreclosing and not foreclosing the downstream firm is affected when two integrated firms are present. When firm $i \in \{1, 2\}$ considers whether to start supplying firm d at price w on the upstream market, it trades off four effects:

- The upstream profit effect.
- The cannibalization effect, which can be decomposed into two components. When firm i starts supplying firm d, it loses some customers to the downstream firm: this is the direct cannibalization effect. Since prices are strategic complements, firm i has incentives to decrease its downstream price following firm d's entry; firm $j \neq i$ reacts by decreasing its price as well: this is the strategic cannibalization effect.
- The softening effect:[39] firm i has incentives to increase its downstream price to preserve its upstream profits. This gives incentives to firm j to raise its price as well under strategic complementarity.

strategies. We do this rigorously in Appendix A.13.

[39] Although we use the same terminology, this softening effect is not exactly the same as the softening effect in Section 3. In Section 3, the softening effect refers to the fact that the upstream supplier charges a higher downstream price than its integrated rival. By contrast, in this section, it refers to the fact that downstream competition is softened by the entry of firm d, *ceteris paribus*, since firm i's best response function shifts upwards.

- The reaction effect: integrated firm $j \neq i$ reacts to the entry of firm d by pricing more aggressively on the downstream market.

Notice that the softening effect and the reaction effect were not present in the upstream bottleneck benchmark. They have an opposite impact on the decision to foreclose. Because of them, the proof of Proposition 10, cannot be adapted to our framework with two integrated firms. In the upstream bottleneck benchmark, the integrated firm can control the strength of the cannibalization effect and earn upstream profits by setting a high input price. However, in a framework with two integrated firms, nothing ensures that the upstream supplier will be able to control for the strength of the reaction effect.

To go further, and to obtain some predictions on the determinants of complete foreclosure, we need to specify the demand functions. We would like to perform comparative statics on three parameters: downstream differentiation, firm d's cost (dis-)advantage, and input differentiation. If we were only interested in downstream differentiation and the downstream firm's efficiency, we could simply use the standard Shubik and Levitan (1980) demand functions. But as we will see, input differentiation is also an important determinant of complete foreclosure. As Ordover and Shaffer (2007), we say that the input is differentiated when firm d's product is a closer substitute to its upstream supplier's product, than to the product of the other integrated firm. There is no parameter to capture this effect in the Shubik and Levitan (1980) demand functions.

In the telecommunications industry, input differentiation may come from geographical coverage considerations. Consider that firms 1 and 2 are two facility-based operators, and assume that their networks only cover a fraction of the territory. If firm d obtains access to the network of firm 1, then, it inherits the geographical coverage of its upstream supplier. This implies that the downstream firm will be able to target all the customers of firm 1, whereas it will not be able to sell to the consumers covered by firm 2's network, and not by firm 1's. Put differently, firm d will be a closer competitor to firm 1. Input differentiation may also come from more technical considerations. For instance, in the video games market, a game developed with a certain 3D engine will be closer, at least graphically, to other games developed with the same engine.

In the following, we build a demand system to capture this effect. We will have to distinguish between $q_k^{(1)}(p)$, the demand received by firm $k \in \{1, 2, d\}$ when firm 1 is the upstream supplier, and $q_k^{(2)}(p)$, firm k's demand when firm 2 supplies the input. As a first step, we look for systems of duopoly $q_i^D(p_1, p_2)$, $i = 1, 2$, and triopoly $q_k^{(i)}(p)$, $i = 1, 2$ and $k = 1, 2, d$, demand functions that satisfy the following properties:

1. Demands are linear, products are substitute and the total demand is non-increasing in prices.
2. Duopoly demands are symmetric: $q_1^D(p_1, p_2) = q_2^D(p_2, p_1)$.
3. Integrated firms are *ex ante* symmetric: $q_d^{(1)}(p_1, p_2, p_d) = q_d^{(2)}(p_2, p_1, p_d)$, $q_1^{(1)}(p_1, p_2, p_d) = q_2^{(2)}(p_2, p_1, p_d)$ and $q_1^{(2)}(p_1, p_2, p_d) = q_2^{(1)}(p_2, p_1, p_d)$.
4. As p_d increases, the industry goes continuously from triopoly to duopoly: if p_d is such that $q_d^{(i)}(p_1, p_2, p_d) = 0$, then, $q_j^D(p_1, p_2) = q_j^{(i)}(p_1, p_2, p_d)$, $i, j \in \{1, 2\}$.

5. Duopoly and triopoly demands are derived from a representative consumer's program, with a quasilinear utility, and a quadratic and strictly concave subutility.

Linearity and symmetry conditions are required essentially for the sake of tractability. Properties 4 and 5, on the other hand, are more crucial, as they ensure that our duopoly and triopoly demand systems are consistent. The following lemma gives a complete characterization of the demand functions satisfying these four properties:

Lemma 4. *Up to a renormalization of prices and quantities, the demand functions that satisfy properties* $1-5$ *can be written as:*

$$\begin{aligned} q_i^D(p) &= 1 - p_i + \sigma p_j \\ q_i^{(i)}(p) &= 1 - p_i + \sigma p_j - \frac{\phi(1+x)}{\beta} q_d^{(i)}(p), \\ q_j^{(i)}(p) &= 1 - p_j + \sigma p_i - \frac{\phi(1-x)}{\beta} q_d^{(i)}(p), \\ q_d^{(i)}(p) &= \alpha - \beta p_d + \phi(1+x)p_i + \phi(1-x)p_j, \end{aligned}$$

$i \neq j$ *in* $\{1,2\}$, $p \geq 0$, *where* $0 < \sigma < 1$, $0 < \alpha\phi(1+|x|)/\beta < 1$, $\beta > 0$, $-1 < x < 1$, $\phi > 0$, $\sigma > \phi^2(1-x^2)/\beta$, *and*

$$0 < \phi(1+|x|)(1 - \frac{2\phi}{\beta}) < 1 - \sigma.$$

Proof. See Appendix A.14. □

The demand functions given in Lemma 4 enable us to parameterize both upstream and downstream differentiation. σ is the usual downstream differentiation parameter when firm d is not active. x is a parameter for input differentiation. When $x = 0$, firm d cannibalizes as much its upstream supplier's demand as the demand of the other integrated firm. By contrast, if x is positive, the downstream firm steals more customers from its supplier, whereas the other integrated firm suffers less from cannibalization. Besides, a change in firm d's downstream price has a stronger impact on its upstream supplier's demand. Conversely, if x is negative, the downstream firm cannibalizes more the consumers of the other integrated firm. Since we are interested in input differentiation, we will restrict the analysis to situations in which x is non-negative.

There are three other parameters in the demand functions given by Lemma 4, that introduce additional sources of asymmetries between the three firms: α, which represents the size of firm d; β, which parameterizes the price sensitivity of firm d's demand; ϕ, which parameterizes the average substitutability between the entrant's product and the integrated firms' products. For the sake of tractability, to get rid of these parameters, in which we are not particularly interested, we require that the demands satisfy the following additional property:

6. Demands are symmetric when the input differentiation parameter is set to 0.

The following lemma characterizes the demand systems that satisfy properties 1 through 6:

Lemma 5. *Up to a renormalization of prices and quantities, the demand functions that satisfy properties* $1-6$*, can be written as:*

$$\begin{aligned} q_i^D(p) &= 1 - p_i + \sigma p_j \\ q_i^{(i)}(p) &= 1 - p_i + \sigma p_j - \frac{\sigma}{\sigma+1}(1+x)q_d^{(i)}(p), \\ q_j^{(i)}(p) &= 1 - p_j + \sigma p_i - \frac{\sigma}{\sigma+1}(1-x)q_d^{(i)}(p), \\ q_d^{(i)}(p) &= \frac{\sigma+1}{2\sigma+1}\left(1 - (\sigma+1)p_d + \sigma(1+x)p_i + \sigma(1-x)p_j\right), \end{aligned}$$

$i \neq j$ *in* $\{1,2\}$*,* $p \geq 0$*, where* $0 < \sigma < 1$ *and* $0 \leq x < 1$*.*

Proof. Immediate. □

We can now use these demand functions to perform comparative statics on the entrant's efficiency, as well as on upstream and downstream differentiation. We obtain the following proposition:

Proposition 11. *Consider the demand functions defined in Lemma 5 with linear downstream costs:*

- *When* $x = 0$ *and* $c_d = c_1 = c_2$*, there exists a threshold* $\sigma^{(\emptyset)}$*, such that complete foreclosure arises in equilibrium if, and only if,* $\sigma \geq \sigma^{(\emptyset)}$*.*
- *When* $c_d = c_1 = c_2$*, there exists a threshold* $x^{(\emptyset)}$ *and a strictly increasing function* $x \in [0, x^{(\emptyset)}] \mapsto \sigma_x^{(\emptyset)}(x)$*, such that complete foreclosure arises in equilibrium if, and only if,* $x \leq x^{(\emptyset)}$ *and* $\sigma \geq \sigma_x^{(\emptyset)}(x)$*.*
- *When* $x = 0$*, there exists a strictly decreasing function* $\sigma \in [0,1] \mapsto c_d^{(\emptyset)}(\sigma)$*, such that complete foreclosure arises in equilibrium if, and only if,* $c_d \geq c_d^{(\emptyset)}(\sigma)$*.*

Proof. We derive these results using numerical simulations. A mathematica file with all the computations is available online. □

Several remarks are in order. Notice first that complete foreclosure is more likely to arise when final products are stronger substitutes. We know from Proposition 10 that this result cannot come from the cannibalization effect alone, since, in the upstream bottleneck benchmark, the upstream supplier was able to control this effect through a high enough input price. Therefore, the explanation must come from an additional effect, which was not present in the upstream bottleneck benchmark, and which affects negatively the upstream supplier's profit: the reaction effect. When the downstream good is poorly differentiated, the upstream supplier suffers more from its integrated rival's aggressive behavior. It may therefore prefer to exclude the downstream firm from the final market. Conversely, when final products are strongly differentiated, the reaction effect is weak, and foreclosure is not profitable.

The fact that complete foreclosure can arise when the downstream firm is as efficient as its integrated rivals is also worth noticing. When $c_d = c_1$, condition (7) holds, and Proposition 10 implies that foreclosing the entrant is not profitable in the upstream bottleneck benchmark. This means that, when final products are strong substitutes, an additional integrated firm can actually make complete foreclosure more likely. Once again, a factor which, in a single-market analysis, would usually imply more competition and therefore lower prices, can have the opposite effect once interactions between the upstream market and the downstream market are taken into account.

As in Section 3, an increase in the downstream firm's marginal cost tends to decrease the potential profits that can be made over input sales. Therefore, supplying the entrant becomes less profitable, and foreclosure becomes more likely.

More surprisingly, when final products are strong substitutes, an increase in the input differentiation parameter tends to decrease the scope for complete foreclosure. While common sense suggests that an integrated firm should be reluctant to supply a downstream buyer, whose product will cannibalize aggressively its downstream sales, input differentiation actually reduces the incentives for complete foreclosure. It is true that an increase in the input differentiation parameter strengthens the cannibalization effect; nevertheless, it also tends to weaken the reaction effect, since the integrated rival is less affected by the entry of firm d. In our setting, the latter effect always dominates the former, so that complete foreclosure does not arise when the input is sufficiently differentiated.

This finding contrasts sharply with Ordover and Shaffer (2007), who show that input differentiation, or, in their terminology, own-supplier cannibalization, leads to complete foreclosure. Ordover and Shaffer (2007)'s result may come from the particular demand system they use. In triopoly, their demand functions can be seen as an alteration of the standard Shubik and Levitan (1980) linear demands. In particular, when there is no input differentiation, the demand of firm k can be written as $q_k = \frac{1}{3}(1 - p_k - \gamma(p_k - \frac{\sum_{k'} p_{k'}}{3}))$. When the downstream firm is not active, they define the duopoly demands as $q_k^D = \frac{1}{2}(1 - p_k - \gamma\frac{p_k - p_{k'}}{2})$. As pointed out by Höffler (2008), this demand system is problematic, since the duopoly and triopoly demands are not derived from the same representative consumer. More importantly, the duopoly is not the limit case of the triopoly: this demand system violates the two consistency properties that we required in Lemmas 4 and 5. This implies that the total demand sometimes behaves in an undesirable way when firm d enters the final market. To see this, suppose that both integrated firms set the same downstream price: $p_1 = p_2 = p$. In this case, the downstream demand is equal to $1-p$. Now, if firm d manages to enter and sets a price $p_d > p$, and if the integrated still set downstream price p, then, the total demand becomes $1 - p - (p_d - p)/3 < 1 - p$. In other words, the total demand decreases when firm d enters the final market, even though both integrated firms keep their downstream prices constant.

This behavior of the total demand has strong implications on the decision to foreclose the entrant: when the input is differentiated, if firm d manages to enter, then, the total demand tends to decrease, and the upstream supplier is more affected by this demand shrinkage. Because of this, with Ordover and Shaffer (2007)'s demands, input differentiation tends to favor complete foreclosure. Because we have imposed consistency requirements 4 and 5, our demand system is not subject to such criticisms, and therefore, we are more confident in the results stated in Proposition 11.

Overall, as in the previous sections, our results point to the following warnings. Factors that traditionally lead to tougher competition in single-market analyses, such as lower differentiation or higher number of firms, may have the opposite effect when the market of interest is an input market which is essentially populated with vertically integrated firms.

7 Endogenous Market Structure

We now endogenize the market structure. We are particularly interested in two questions. Can a market structure in which the input market is only populated with integrated firms emerge endogenously? Would a horizontal merger between an integrated firm and an unintegrated downstream firm be profitable in our framework?

7.1 Anticompetitive Vertical Mergers

In this section, we consider that the industry is initially disintegrated. There are two unintegrated upstream firms, $U1$ and $U2$, and three unintegrated downstream firms, $D1$, $D2$ and $D3$. If firms Ui and Dk merge, we call $UiDk$ the merged firm. Following Ordover, Saloner, and Salop (1990)'s seminal paper, we adopt the following timing:

Stage 1 – Merger: Downstream firms can bid to acquire upstream firm $U1$.

Stage 2 – Counter-merger: If a vertical merger has taken place in the previous period, the remaining unintegrated downstream firms can bid to acquire firm $U2$.

Stage 3 – Upstream competition: Each upstream firm (integrated or not) announces the price at which it is ready to supply any unintegrated downstream firm. Each downstream firm elects at most one upstream supplier.

Stage 4 – Downstream competition: Downstream firms (integrated or not) set their prices on the downstream market. Then, unintegrated downstream firms are allowed to switch to another supplier at zero cost, if this is strictly profitable to do so.

We look for the subgame-perfect equilibria of this four-stage game. To avoid trivial situations, we assume that upstream firms accept to merge only if they receive at least one strictly positive bid. To simplify the exposition, we assume that downstream costs are linear, and that the downstream firms are symmetric. Apart from that, we make the same assumptions as in Section 2, regarding the demand functions: demands satisfy the usual monotonicity conditions, in all period 4 subgames, downstream best responses as well as the downstream equilibrium are unique, and downstream prices are strategic complements. For conciseness, we solve the model under the assumption that downstream firms have access to an alternative source of input, although similar results could be derived if complete foreclosure were feasible.

Notice that we assume that upstream firms are not able to discriminate between downstream buyers, when they set their upstream prices. This assumption enables us to rule out some pathological cases in the one-merger subgame.[40]

[40] Assuming that offers are non-discriminatory allows us to get rid of the following equilibrium candidate. Suppose that discrimination is allowed on the input market, and consider the subgame in which firm $U1$ has merged with

In the previous sections, we have already characterized the equilibrium outcomes in the two-merger subgame. To find the equilibria of our four-stage game, all we need to do now is analyze the zero and one-merger subgames. We prove the following lemma:

Lemma 6. *When no merger or one merger has taken place, the downstream firms purchase the input at marginal cost in equilibrium.*

Proof. The proof for the zero-merger subgame is immediate, as it is just the single-market Bertrand result. The main argument leading to marginal cost pricing in the one-merger subgame can be found in Chen (2001) (when upstream switching costs and upstream cost differentials are set to zero). We extend Chen (2001)'s result to our framework with an integrated firm, an unintegrated upstream firm and two downstream buyers in Appendix A.15. □

When the industry is disintegrated, the standard Bertrand logic can be applied, and upstream firms undercut each other until the marginal cost is attained. When one-merger has taken place, say, between upstream firm $U1$ and downstream firm $D1$, upstream competition still yields marginal cost pricing. Intuitively, if firm $U1-D1$ supplies the upstream market with a positive markup, firm $U2$ clearly wants to undercut, since the input market is its sole source of profit. If firm $U2$ is the upstream supplier, then the integrated firm has even more incentives to undercut: if firm $U1-D1$ corners the input market, it makes upstream profits, and it creates a softening effect, which relaxes downstream competition.

Lemma 6 highlights the fact that upstream competition between integrated firms tends to be less intense than competition between unintegrated upstream firms, or competition between unintegrated upstream firms and integrated firms. In a nutshell, a non-integrated upstream firm always wants to undercut its rivals, integrated or not. A vertically integrated firm has even stronger incentives to undercut its non-integrated rivals. By contrast, an integrated firm may not want to undercut an integrated rival in order not to lose the softening effect.

We introduce the following notations: in the two-merger subgame, $\Pi_{US}(w)$, $\Pi_{IR}(w)$ and $\Pi_D(w)$ are the profits of the upstream supplier, the integrated rival, and the downstream firm respectively, when the upstream market is supplied at price w; Π^* is the equilibrium profit of a downstream firm, integrated or not, when all downstream firms obtain the input at marginal cost.[41] As before, the monopoly upstream price, w_m, maximizes $\Pi_{US}(w)$. Putting Proposition 2 and Lemma 6 together, we obtain the following proposition:

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firm $D1$, and no other mergers have taken place. Firm $U1-D1$ offers its monopoly upstream price to firm $D2$, and makes no offer to firm $D3$. Similarly, firm $U2$ offers its monopoly upstream price to firm $D3$, and makes no offer to firm $D2$. Firm $U2$ prefers not to undercut the input price offered to firm $D2$, since firm $U1-D1$ would then become more aggressive on the final market, which would lower the input demand of firm $D3$. Firm $U1-D1$ does not want to supply firm $D3$ either, since if it did so, it would become less aggressive on the final market; by strategic complementarity, firm $D2$ would increase its price as well, which may lower its input demand. This situation seems rather unlikely, and it does not arise with the specific demand functions used in this article, but there is no obvious way to rule it out with general demand functions when input price discrimination is allowed.

[41] Since downstream firms are symmetric, these profit functions are well-defined. Notice that, using the notations introduced in Section 2, $\Pi_{US}(w) = \pi_1^{(1)}(w)$, $\Pi_{IR} = \pi_2^{(1)}(w)$ and $\Pi_D(w) = \pi_d^{(1)}(w)$. Notice also that $\Pi_k(m) = \Pi^*$, for $k = US, IR, D$.

Proposition 12. *If $\Pi_{US}(w_m) \leq \Pi_{IR}(w_m)$, and if integrated firms*

- *do not play weakly dominated strategies on the upstream market*
- *or do not play equilibria that are Pareto-dominated by another equilibrium,*

then, in equilibrium, there are two mergers and the upstream market is supplied at the monopoly price.

Proof. See Appendix A.16. □

When the softening effect is strong enough, firms merge vertically for purely anticompetitive reasons, namely, to implement a monopoly-like equilibrium.[42] Notice that our result is robust to the Chicago School criticism, which, in our framework, could be stated as follows: vertical mergers with $U1$ and $U2$ cannot be anticompetitive, since vertical integration does not annihilate the competitive pressure on the input market; if an integrated firm supplies the input above marginal cost, then the other integrated firm should set a slightly lower price, to capture the upstream market, without changing the downstream competition outcome. But we know that the downstream outcome is actually strongly affected when a firm undercuts.

Now, it is interesting to apply our model to the TomTom / Tele Atlas and Nokia / NAVTEQ merger cases, which we mentioned in the introduction. In both cases, the European Commission considered that mobile phones and personal navigation devices were not part of the same product market. This assumption is hardly debatable, at least in the short run, and it implies that these two vertical mergers should no generate anticompetitive effects, as shown in Proposition 3. However, as pointed out in European Commission (2008), "the Commission did not exclude that, as technology evolves, both markets will increasingly converge". Put differently, it may be that the substitutability between TomTom's and Nokia's products will rise over time. If this indeed happens, then our model predicts that, while partial foreclosure is not an issue in the short run, it may become more of a concern in the long run.

Our results also have interesting implications for the regulation of the broadband market. Viviane Reding, Member of the European Commission responsible for Information Society and Media, has many times argued that structural separation of the dominant operator, i.e., the separation of the incumbent into a wholesale and a retail unit, was a policy option.[43] In our model, upstream competition between integrated firms is softer than competition between integrated firms and upstream firms, and the vertical separation of a vertically integrated firm can therefore shift the industry from a monopoly-like outcome to the perfect competition outcome.

[42]We do not discuss the bids that lead to this equilibrium merger wave, since they depend heavily on the assumption that upstream firms are given all the bargaining power in the merger game.

[43]In May 2007, in a speech, she declared: "I believe that functional separation (...) could indeed serve to make competition more effective in a service-based competition environment where infrastructure-based competition is not expected to develop in a reasonable period. It may be a useful remedy in specific cases. It is certainly not a panacea." (Viviane Reding, "How Europe can Bridge the Broadband Gap", Brussels, 14 May 2007).

Notice that, if we drop the equilibrium selection criteria stated in Proposition 12, other equilibria of the mergers game can exist. For instance, since the Bertrand outcome is always an equilibrium of the upstream competition subgame when two mergers have taken place, it is easy to construct a subgame-perfect equilibrium with no merger.

A more interesting result is that pro-competitive one-merger equilibria can also exist. To see this, assume that $\Pi_{US}(w_m) \leq \Pi_{IR}(w_m)$, so that monopoly-like equilibria exist, but assume that these equilibria are not played in all the two-merger subgames. More precisely, assume that a monopoly-like equilibrium, where $U1 - D1$ is the upstream supplier, is played if $D1$ merges with $U1$ and $D2$ merges with $U2$, whereas the Bertrand equilibrium prevails in all other two-merger subgames. Then, $D3$ may want to merge in the first stage to avoid a wave of mergers involving $D1$ and $D2$ that would lead to its partial foreclosure. Such a one-merger outcome is an equilibrium under the following conditions. First, if $D1$ wins the first stage auction, then $D2$ wins the second stage auction. This occurs provided that $D2$'s gains from merging are larger than $D3$'s losses from not merging: $\Pi_{IR}(w_m) - \Pi^* \geq \Pi^* - \Pi_D(w_m)$. Second, $D3$ wins the first stage auction, which occurs when its losses from not merging are larger than $D1$'s gains from merging: $\Pi^* - \Pi_D(w_m) \geq \Pi_{US}(w_m) - \Pi^*(m)$.[44] It can be shown that these conditions hold in the linear example of Section 3.2 with concave downstream costs.

In antitrust parlance, firm $D3$ is a maverick competitor: it will never accept to implement a non-competitive equilibrium. If the maverick is sufficiently harmed when its rivals merge and implement a partial foreclosure equilibrium, it can vertically integrate to ensure tough competition on the upstream market. In that case, the potential maverick becomes an effective maverick by preventing an anticompetitive wave of mergers.

We now discuss how Proposition 12 is affected when firms are allowed to offer two-part tariffs on the upstream market. As in Section 3.4, we need to assume that downstream firms are not allowed to switch to another supplier at the end of stage 4, in order to simplify the analysis of the upstream suppliers' choice game.

Assume first that the fixed part of the tariff can be negative or positive. Then, we know from Proposition 5 that partial foreclosure equilibria always exist in the two-merger subgame. Unfortunately, the subgames with zero or one merger introduce some important complications. In the no-merger subgame, it can be shown that there exists no pure-strategy subgame-perfect equilibrium.[45] In the one-merger subgame, in the spirit of Chen and Riordan (2007), there exists an

[44] There exist one-merger equilibria under weaker conditions, but they are supported by less natural anticipations schemes. For instance, if $D1$ wins the first auction, and if $D2$ or $D3$ wins the subsequent auction, then a monopoly-like outcome is implemented and $U1 - D1$ is the upstream supplier; in all other two-merger subgames, the Bertrand equilibrium is played. $D2$ and $D3$ have a lot to lose if they let $D1$ win the first auction, since they have to engage in a fierce bidding war in the subsequent auction to avoid partial foreclosure. As a result they have incentives to prevent the first merger. They can do so if the losses they incur following the first merger are higher than the gain captured by firm $D1$: $\Pi^* - \Pi_D(w_m) \geq \Pi_{US}(w_m) - \Pi^*$. Condition $\Pi_{IR}(w_m) - \Pi^* \geq \Pi^* - \Pi_D(w_m)$ does need to hold.

[45] For conciseness, we do not give the formal proof of this negative result in this paper. Broadly speaking, if a pure strategy equilibrium existed, the variable parts of the tariffs would have to satisfy the following properties. First, they should be best responses to one another; for instance, if $U1$ supplies $D1$ and $U2$ supplies $D2$ and $D3$, then the variable part that $U1$ offers to $D1$ should maximize the joint profit of these two firms, taking the other offers as given.

equilibrium in which the integrated firm sets a variable part above marginal cost, which maximizes the sum of its profits and the profits of the downstream firms, and a negative fixed part which prevent downstream firms from purchasing from the unintegrated upstream firm. Given these complications, it is not clear whether the full integration result stated in Proposition 12 extends to two-part tariff competition, when negative fixed fees are allowed.

By contrast, when negative fixed fees are forbidden, it is easy to show that the Bertrand outcome is always an equilibrium of the zero and one-merger subgames, under the reasonable assumption that a downstream firm's profit is decreasing in the price at which it purchases the input. In the two-merger subgame, we know from Proposition 5 that partial foreclosure equilibria exist if $\Pi_{US}(w_{tp}) \leq \Pi_{IR}(w_{tp})$, namely, if the softening effect is strong enough. When this is the case, there exists an equilibrium with two mergers.

7.2 Vertical Mergers and Efficiency Gains

In the previous subsection, vertical mergers arised in equilibrium for purely anticompetitive reasons. However, efficiency effects play an important role in the assessment of the overall impact of vertical mergers on social welfare and consumers' surplus. In most non-horizontal merger guidelines, competition authorities commit to balancing the potential anticompetitive effects and efficiency gains when deciding whether to challenge a vertical merger.[46] In this section, we analyze the impact of efficiency gains from vertical integration on social welfare and consumers surplus in our equilibrium model of vertical mergers. We are interested in two types of efficiency gains: downstream synergies and upstream synergies.

Downstream synergies Consider first that a vertical merger reduces the downstream marginal cost of the newly integrated firm. More precisely, assume that integrated firms operate with downstream marginal cost $c - \delta$ ($\delta > 0$), whereas unintegrated downstream firms' marginal cost is c. Notice that Lemma 6 still applies, as we did not use the fact that downstream firms had the same downstream marginal cost to prove it in Section 7.1. As a result, when at most one merger has taken place, the input is priced at marginal cost in equilibrium. Denote by $\Pi^*(c_i, c_j, c_k)$ the profit of firm i at the downstream equilibrium, when its downstream marginal cost is c_i, its rivals' marginal costs are c_j and c_k, and all the firms purchase (or produce) the input at marginal cost m. With this notation, the profit of an unintegrated downstream firm in the zero-merger and one-merger subgames is $\Pi^*(c, c, c-\delta)$, whereas the profit an integrated firm in the one-merger subgame is given by $\Pi^*(c-\delta, c, c)$. We make the reasonable assumption that a firm's profit decreases in its own cost ($\partial\Pi^*/\partial c_i < 0$), increases in its rivals' costs (($\partial\Pi^*/\partial c_j,\ \partial\Pi^*/\partial c_k > 0$)), and decreases when all costs increase: $\partial\Pi^*/\partial c_i + \partial\Pi^*/\partial c_j + \partial\Pi^*/\partial c_k < 0$.

When two mergers have taken place, there are two vertically integrated firms competing on the upstream market. In this case, we still denote by $\Pi_{US}(w)$, $\Pi_{IR}(w)$ and $\Pi_D(w)$ the profits of the

Second, they should maximize the industry profit. Unless the final products are independent, these conditions cannot hold simultaneously.

[46]See, for instance, EC (2007).

three firms, where we omit the downstream cost parameters to keep simple notations. We know from Proposition 2 that monopoly-like equilibria exist if, and only if, $\Pi_{US}(w_m) \leq \Pi_{IR}(w_m)$. We also know that monopoly-like equilibria Pareto-dominate all other equilibria, from the integrated firms' viewpoint. All other equilibria, including the Bertrand equilibrium, feature both integrated firms setting the same upstream price $w \leq w_m$, and earning the same profit: $\Pi_{US}(w) = \Pi_{IR}(w)$. As pointed out in Section 3.1, there may be supra-competitive symmetric equilibria, with $w < m$. However, if firms do not coordinate on Pareto-dominated equilibria, which we assume in the following, these equilibria are never played, since they are dominated by the Bertrand equilibrium. This implies that, in the two-merger subgame, integrated firms earn at least $\Pi_{US}(m) = \Pi_{IR}(m) = \Pi^*(c-\delta, c-\delta, c)$. Given our assumptions on function Π^*, we can conclude that there will always be two mergers in equilibrium, due to the efficiency gains.

Because of these synergies, it is no longer clear whether competition authorities should challenge vertical mergers, since there may now be a tradeoff between efficiency and foreclosure effects. A related question is whether competition authorities should be more favorable to vertical mergers when downstream synergies are stronger. On the one hand, larger efficiency gains make vertical mergers more desirable both for consumers and in terms of industry welfare, *for a given outcome of upstream competition in the two-merger subgame.* On the other hand, we also know from Proposition 4 that a higher downstream cost disadvantage of the unintegrated downstream firm tends to favor the emergence of partial foreclosure equilibria.

Consider a competition authority, that seeks to maximize social welfare or consumers' surplus, and suppose that it can intervene at the end of periods 1 and 2 to forbid the first merger or the counter-merger. From an antitrust perspective, the first merger is always beneficial, since it only creates efficiency effects. Now, the question is whether the second one should be challenged.

To deal with this issue, we assume that there is a representative consumer in the industry, whose preferences can be written as follows:

$$U = q_0 + \sum_{k=1}^{3} q_k - \frac{1}{2}\left(\sum_{k=1}^{3} q_k\right)^2 - \frac{3}{2(1+\gamma)}\left(\sum_{k=1}^{3} q_k^2 - \frac{\left(\sum_{k=1}^{3} q_k\right)^2}{3}\right), \tag{9}$$

where q_0 is consumption of the numeraire, q_k denotes consumption of firm Dk's product, $k = 1, 2, 3$, and $\gamma > 0$. This yields the usual Shubik and Levitan (1980) demand functions:

$$q_i = \frac{1}{3}\left(1 - p_i - \gamma\left(p_i - \frac{\sum_{k=1}^{3} p_k}{3}\right)\right). \tag{10}$$

Before stating the result we notice that, after two vertical mergers, the remaining downstream firm might be squeezed from the market if it is too inefficient relatively to integrated firms or if the upstream price is too high. In particular, as shown in the appendix, there exists a threshold $w_{\max}(\gamma, \delta)$, such that $\Pi_D(w) = 0$ whenever $w \geq w_{\max}(\gamma, \delta)$. Besides, $\Pi_{US}(w)$ is strictly concave in w and reaches its maximum at $w_m(\gamma, c) < w_{\max}(\gamma, \delta)$ if, and only if, δ is strictly below a threshold value $\delta_{\max}(\gamma)$. In the following, we restrict our attention to $\delta < \delta_{\max}(\gamma)$, and we assume that the alternative source of input is not too efficient, so that $w_m(\gamma, \delta) \leq \overline{m} < w_{\max}(\gamma, \delta)$, to ensure that the monopoly upstream price is $w_m(\gamma, \delta)$.

The following proposition summarizes the impact on social welfare and consumers' surplus of the second merger:

Proposition 13. *When integrated firms do not play Pareto-dominated equilibria, there are two vertical mergers in equilibrium. Moreover, when preferences and demands are given by equations* (9) *and* (10)*, there exist thresholds* $0 \leq \overline{\delta}_M(\gamma) \leq \overline{\delta}_W(\gamma) \leq \overline{\delta}_{CS}(\gamma)$.[47] *These thresholds are decreasing in* γ*, and partition the* (γ, c) *plane into four areas:*

- *(Area 1): If* $\delta < \overline{\delta}_M(\gamma)$*, then the second merger leads to a Bertrand outcome on the upstream market and increases consumers' surplus and social welfare,*
- *(Area 2): If* $\delta \in [\overline{\delta}_M(\gamma), \overline{\delta}_W(\gamma)]$*, then the second merger leads to a monopoly-like outcome on the upstream market and reduces consumers' surplus and social welfare,*
- *(Area 3): If* $\delta \in [\overline{\delta}_W(\gamma), \overline{\delta}_{CS}(\gamma)]$*, then the second merger leads to a monopoly-like outcome on the upstream market, reduces consumers' surplus, and increases social welfare,*
- *(Area 4): If* $\delta \geq \overline{\delta}_{CS}(\gamma)$*, then the second merger leads to a monopoly-like outcome on the upstream market and increases consumers' surplus and social welfare.*

Besides, there exist $0 < \underline{\gamma}_{CS} < \overline{\gamma}_{CS} < \underline{\gamma}_W < \overline{\gamma}$*, such that area 1 is empty when* $\gamma \geq \overline{\gamma}$*, area 2 is empty when* $\gamma \leq \underline{\gamma}_W$*, area 3 is empty when* $\gamma \geq \underline{\gamma}_{CS}$*, and area 4 is empty when* $\gamma \leq \overline{\gamma}_{CS}$*.*

Proof. These results are obtained by running numerical simulations. A mathematica file detailing the computations is available online. □

The four areas defined in Proposition 13 are depicted in Figure 5. In area 1, downstream competition is not too fierce, or downstream synergies are not too important. In this case, we know from Propositions 3 and 4 that a merger wave does not lead to partial foreclosure. Therefore, the second merger improves both social welfare and consumers' surplus, due to the direct efficiency effect. In area 2, downstream competition is tough, which implies, from Proposition 3 that the second merger creates a foreclosure effect, and synergies are weak. The efficiency effect is therefore dominated by the foreclosure effect, and the second merger is detrimental to consumers and to the welfare of the industry. In area 3, synergies are stronger than in area 2, and therefore, the efficiency effect dominates the foreclosure effect for social welfare, but not for consumers' surplus. In area 4, downstream competition is mild, and monopoly-like equilibria exist mainly because downstream synergies are important. As a result, the efficiency effect dominates the foreclosure effect.

An interesting consequence of Proposition 13 is that, for a given degree of downstream substitutability, the optimal response of the competition authority to the second merger is not necessarily

[47] M, W and CS stand for monopoly-like, welfare and consumers' surplus, respectively.

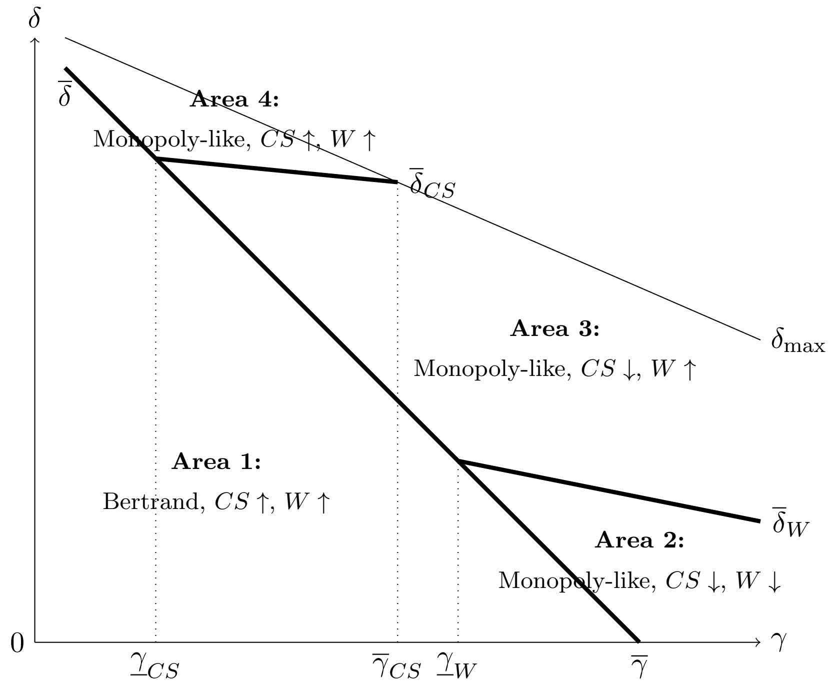

Figure 5: Welfare effects of the second merger with downstream synergies

monotonic in the strength of downstream synergies. In particular, the simple rule of thumb, according to which the competition authority should be more favorable to a vertical merger when downstream synergies get stronger may not be accurate. This is because stronger synergies strengthen the efficiency effect of the second merger, but they may also create a foreclosure effect. Consider for instance that γ is between $\underline{\gamma}_W$ and $\overline{\gamma}$. Then, a welfare-oriented competition authority should give clearance to the second merger when $\delta < \overline{\delta}$ or $\delta > \overline{\delta}_W$, and forbid it when $\overline{\delta} \leq \delta \leq \overline{\delta}_W$. Similarly, a competition authority biased towards consumers should clear the second merger if, and only if, $\delta < \overline{\delta}$.

Upstream synergies We now consider that a vertical merger creates upstream synergies, that reduce the upstream marginal cost from m to $m - \delta < m$. As long as an integrated firm produces the intermediate input for its own downstream division only, the difference between upstream and downstream efficiency gains is immaterial. The difference is that upstream efficiency gains also lower the cost of producing the input for other downstream firms.

It is then straightforward to extend the analysis we have just made for downstream synergies, to show that, under similar assumptions (a firm's profit decreases in its marginal cost, increases in its rivals' costs, and decreases when all costs increase, firms do not play Pareto-dominated equilibria), two vertical mergers take place in equilibrium. As before, the second merger creates efficiency effects and, potentially, a foreclosure effect, and it is not clear whether the overall impact on welfare is positive. To sort out these effects, let us consider once again the linear demands defined by equations (9) and (10).

To avoid introducing new notations, let us call firm 1 the first vertically integrated firm, and firm 2 the second one. The remaining downstream firm is relabeled as firm d. Define $\tilde{p}_k \equiv (p_k - m + \delta - c)/(1 - m + \delta - c)$, $k = 1, 2, d$ and $\tilde{w} \equiv (w - m + \delta)/(1 - m + \delta - c)$. Notice that the demand received by firm k can be written as follows:

$$\begin{aligned} q_k(p_1, p_2, p_d) &= \frac{1}{3}\left(1 - p_k - \gamma(p_k - \frac{\sum_{k'=1}^{3} p_{k'}}{3})\right), \\ &= (1 - m + \delta - c)\left(1 - \tilde{p}_k - \gamma(\tilde{p}_k - \frac{\sum_{k'=1}^{3} \tilde{p}_{k'}}{3})\right), \\ &= (1 - m + \delta - c) q_k(\tilde{p}_1, \tilde{p}_2, \tilde{p}_d). \end{aligned}$$

This implies that the payoff functions in stage 4 can be rewritten as follows:

$$\begin{aligned} \tilde{\pi}_i^{(i)}(p, w) &= (1 - m + \delta - c)^2 \left\{\tilde{p}_i q_i(\tilde{p}) + \tilde{w} q_d(\tilde{p})\right\}, \\ \tilde{\pi}_j^{(i)}(p, w) &= (1 - m + \delta - c)^2 \left\{\tilde{p}_j q_j(\tilde{p})\right\}, \\ \tilde{\pi}_d^{(i)}(p, w) &= (1 - m + \delta - c)^2 \left\{(\tilde{p}_d - \tilde{w}) q_d(\tilde{p})\right\}. \end{aligned}$$

Therefore, the vector of (normalized) equilibrium downstream prices, $p^{(i)}(w)$ does not depend on m, δ and c. Similarly, the (normalized) upstream monopoly price, $\tilde{w}_m$ depends only on γ. The profit of a firm at the downstream equilibrium can then be written as $(1 - m + \delta - c)^2$ times the downstream equilibrium profit of this firm when m, c and δ are equal to zero. In particular, the comparison between the equilibrium profits of the upstream supplier and its integrated rival does not depend on cost parameters. We can then apply Proposition 3, which tells us that there exists $\overline{\gamma}$, which is independent of m, c and δ, such that partial foreclosure equilibria exist if, and only if, $\gamma \geq \overline{\gamma}$. In other words, the strength of upstream synergies has no impact on the emergence of partial foreclosure equilibria. We can then prove the following proposition:

Proposition 14. *When integrated firms do not play Pareto-dominated equilibria, there are two vertical mergers in equilibrium. Moreover, when preferences and demands are given by equations* (9) *and* (10)*, there exists a threshold $\overline{\gamma}$, and two decreasing function $\overline{\delta}^{CS}(\gamma) < \overline{\delta}^{W}(\gamma)$ such that:*

- *If $\gamma < \overline{\gamma}$, then the second merger leads to the Bertrand outcome, and raises consumers' surplus and welfare.*
- *If $\gamma \geq \overline{\gamma}$, then the second merger leads to a monopoly-like outcome. It raises consumers' surplus (resp. social welfare) if, and only if, $\delta \geq \overline{\delta}^{CS}(\gamma)$ (resp. $\delta \geq \overline{\delta}^{W}(\gamma)$).*

Proof. See Appendix A.17. □

Proposition 14 is depicted on Figure 6. Contrary to downstream synergies, upstream synergies do not increase the scope for partial foreclosure. When δ increases, the efficiency effect rises and the

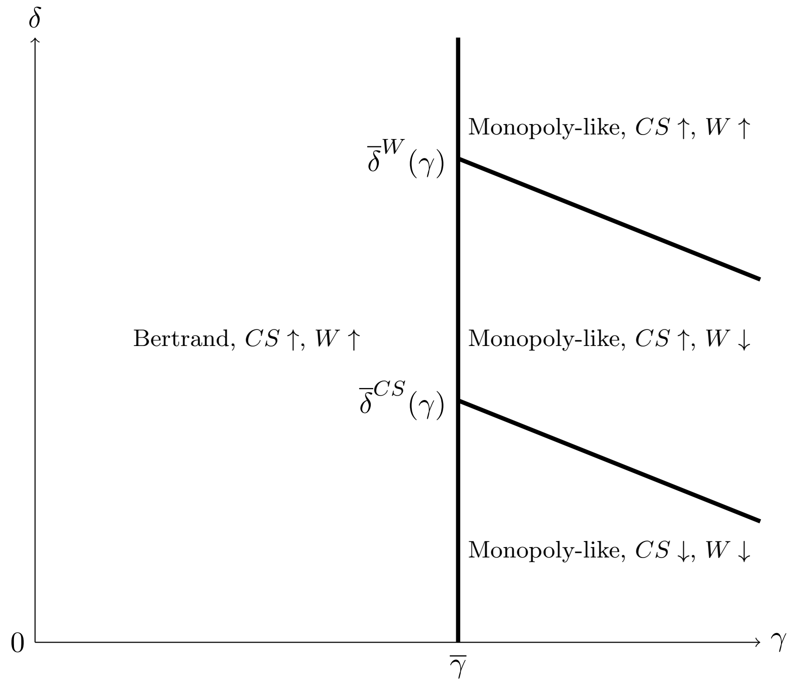

Figure 6: Welfare effects of the second merger with upstream synergies

foreclosure effect is unaffected. Therefore, a simple rule of thumb, where the competition authority clears the second merger when upstream synergies are sufficiently important is accurate.

7.3 Horizontal Mergers with Vertical Aspects

In this section, we start from the integrated industry, as in Section 2, and we aim to answer two questions. First, would a horizontal merger between an integrated firm and the downstream be profitable? Second, would it be socially desirable? We address these questions within the following model:

*Stage 1 – Horizontal merger***:** Integrated firms 1 and 2 can bid to acquire downstream firm d.

*Stage 2 – Upstream competition***:** If firm d remains independent, integrated firms compete with two-part tariffs on the upstream market.[48] Firm d observes the two offers, and elects its upstream supplier.

*Stage 3 – Downstream competition***:** Downstream firms (integrated or not) set their prices on the downstream market.

[48] Similar insights could be derived if we assumed linear tariff competition on the input market. The decision to merge horizontally would still trade off the commitment effect against the price coordination effect. In particular, it is easy to show that, with linear demands, a horizontal merger is not profitable whenever monopoly-like equilibria exist (γ high enough). We choose to present our results with two-part tariffs, as this leads to a more interesting welfare analysis.

Notice that we do not allow a horizontal merger between the two integrated firms. The reason is that, in this framework, such a merger would create a monopoly on the upstream market, and most antitrust authorities would probably not let that happen.

If firm d were able to produce the input in-house with constant marginal cost m, then, a merger between firms 1 and d would always be profitable. As shown by Deneckere and Davidson (1985), in our framework with price competition, product differentiation and strategic complementarity, a horizontal merger allows the merging parties to internalize competitive externalities, and the non-merging firm reacts to the merger by increasing its downstream price. Such a merger would obviously be detrimental to consumers and industry welfare.

However, we know from Proposition 5 that, if firm d remains independent, then upstream competition leads to partial foreclosure in equilibrium. Assume, without loss of generality, that firm 1 is anticipated to become the upstream supplier in the no-merger subgame. If there is no horizontal merger, firm 1 sells the input to firm d with a variable part that maximize their joint profits. Depending on whether the downstream firm's participation constraint or firm 2's incentive constraint binds first, firm 2 will either make as much profits as firm 1, or strictly more profits than firm 1. In both cases, it is therefore sufficient to check whether a horizontal merger increases the joint profit of firms 1 and d to find the equilibrium of this game.

When firms 1 and d are independent, the fact that the input is priced above cost softens downstream competition for two reasons. First, firm d operates with a high marginal cost, which induces it to behave less aggressively. Second, the softening effect raises firm 1's opportunity cost of decreasing its downstream price. Put differently, partial foreclosure provides firms 1 and d with a commitment to be soft competitors on the final market, and we know from Fudenberg and Tirole (1984) that the two firms benefit from this commitment. The only drawback of this strategy is that the variable part of the two-part tariff is a somewhat blunt instrument, which generally does not induce firms 1 and d to maximize their joint profits when they set their downstream prices.

On the other hand, if firms 1 and d merge, then, they set their downstream prices to maximize their joint profit. However, they no longer benefit from the commitment effects which we emphasized in the non-integration case. To summarize, the decision to merge horizontally trades off two effects. On the one hand, a horizontal merger enables the merging parties to coordinate their downstream prices. On the other hand, a two-part tariff with a variable part above cost provides firms with a commitment to behave softly on the downstream market. To sort out the coordination effect and the commitment effect, we use once again the preferences and demands defined in equations (9) and (10) with linear costs. We obtain the following proposition:

Proposition 15. *When preferences and demands are given by equations* (9) *and* (10)*, there exist three thresholds* $0 < \gamma_W < \gamma_{CS} < \gamma_H$*, such that:*

- *If* $0 < \gamma < \gamma_W$ *(resp. if* $0 < \gamma < \gamma_{CS}$*), then a horizontal merger arises in equilibrium, and it degrades social welfare (resp. consumers' surplus).*
- *If* $\gamma_W < \gamma < \gamma_H$ *(resp. if* $\gamma_{CS} < \gamma < \gamma_H$*), then a horizontal merger takes place in equilibrium, and it increases social welfare (resp. consumers' surplus).*

- *If $\gamma > \gamma_W$ (resp. $\gamma > \gamma_{CS}$), firms do not merge in equilibrium, but a horizontal merger would increase social welfare (resp. consumers' surplus).*

Proof. See Appendix A.18. □

To summarize, when γ is low, a merger takes place in equilibrium, and it lowers welfare and consumers' surplus. For intermediate values of γ, a welfare-improving and consumers' surplus-improving merger arises. Last, when γ is large, firms choose not to merge, whereas a merger would be optimal for industry and consumers welfare.

To see the intuition behind Proposition 15, assume that the substitutability parameter is high. A horizontal merger between firms 1 and d allows these firms to internalize the competitive externalities that they exert on each other. But since γ is high, firm $1-d$ has strong incentives to lower both prices p_1 and p_d to steal consumers from firm 2. Put differently, when final products are strong substitutes, the anticompetitive impact of a horizontal merger is weak, since firms $1-d$ and 2 compete head-to-head anyway. However, if firms 1 and d can obtain a commitment not to compete too aggressively, which they can achieve by staying separate and by trading the input under the optimal two-part tariff contract, they can move the industry towards a less competitive outcome. This also explains why welfare and consumers' surplus tend to be higher when firms 1 and d merge, when γ is sufficiently large. Conversely, when γ is low, the merged firm $1-d$ has little incentives to lower its downstream prices. A commitment power on the downstream market is therefore less valuable, and firms 1 and d prefer being able to better coordinate their downstream prices.

Proposition 15 has some interesting implications for intermediate values of the substitutability parameter. When γ is intermediate, the downstream market is rather concentrated, and products are relatively close substitutes. Competition authorities would therefore be tempted to challenge this horizontal merger.[49] Yet, our analysis unveils that, once interactions between upstream and downstream markets are taken into account, this merger would actually improve consumers' surplus and welfare. It is therefore important to consider the vertical aspects of a horizontal merger when assessing its potential anticompetitive impact, since focusing on its horizontal dimensions may lead to misguided antitrust decisions.

8 Conclusion

While competition between vertically integrated firms can be observed in several industries, its implications for regulation and antitrust policy have received little attention in the literature. In this paper, we have developed a model to analyze these issues. Because of the softening effect, according to which an integrated firm charges a higher downstream price when it supplies the upstream market, upstream competition between integrated firms tends to be less intense than competition between integrated firms and upstream firms. This implies that, even if all the usual ingredients of Bertrand competition are present, the monopoly outcome may persist on the input market.

[49] See for instance FTC (1997).

We have shown that, under linear tarif competition, the key determinants leading to equilibrium partial foreclosure are the downstream products' substitutability, and the entrant's cost (dis-)advantage. With two-part tariffs, upstream competition always lead to a partial foreclosure equilibrium with positive upstream profits. The impact of market structure, on the other hand, is usually ambiguous. For instance, while conventional wisdom suggests that the entry of integrated firms should increase the competitive pressure on the upstream market, we have shown that this is not necessarily the case, since such a change in market structure moves the softening effect and the upstream profit effect in the same direction. We have provided an example, with plausible demand functions, in which an increase in the number of integrated firms usually makes partial foreclosure easier to sustain.

We have also obtained some results on the determinants of complete foreclosure. In the upstream bottleneck benchmark, i.e., when only one integrated firm is present, complete foreclosure is an unlikely outcome, since the integrated firm can control the strength of the cannibalization effect by setting a high enough input price, while still earning some upstream profits. With two integrated firms, the tradeoff between foreclosing and not foreclosing is no longer the same, since the integrated rival will react to the entry of the downstream firm. We show that, when this reaction effect is strong enough, which is typically the case when downstream products are close substitutes, complete foreclosure becomes profitable. Again, the entry of integrated firms may actually make the upstream market less competitive. We have also seen that the downstream firm is more likely to be completely foreclosed when it is inefficient, or when the input is not too differentiated.

Since partial foreclosure equilibria are detrimental to consumers and to the industry welfare, regulators or antitrust authorities may want to intervene. We have shown that a price cap can be an efficient means to destroy all partial foreclosure equilibria. We have also argued that the vertical separation of an integrated firm, a policy option that has often been considered by telecommunications regulators, may play the same role, since competition between upstream and integrated firms tends to be tougher than competition between integrated firms.

By challenging vertical merger waves, competition authorities can also avoid the emergence of market structures, in which only vertically integrated firms can produce the input. As a general rule of thumb, antitrust agencies should be more concerned about vertical mergers when firms compete intensively on the downstream market. When vertical integration involves efficiency gains, a similar rule of thumb for the size of these efficiency gains may be misleading, since stronger synergies may also create a foreclosure effect.

We have also derived some insights on the profitability and social desirability of horizontal mergers with vertical aspects. Because of the softening effect, a horizontal merger between an integrated firm and a downstream firm is not necessarily profitable in our setting. For the same reasons, a horizontal merger can also be profitable for the merging parties, and increase welfare. This underlines the fact that vertical dimensions should be taken into account for the assessment of horizontal mergers.

A Appendix

A.1 A Preliminary Lemma

To ease the proofs of all lemmas and propositions, we begin by proving the following technical lemma.

Lemma 7. *If the downstream best response function of at least one firm shifts upwards (downwards), then all equilibrium downstream prices increase (decrease) strictly.*

Proof. Assume that firm k's (and possibly some other firms') best response shifts upwards. Since the second order condition holds, this happens if, and only if, the first derivative of its profit with respect to its price shifts upwards. For all l in $\{1, 2, d\}$, let us denote by $\phi_l^{(0)}(p)$ (respectively $\phi_l^{(1)}(p)$) the profit of firm l before (resp. after) the marginal profit shift. By strategic complementarity, the game defined by payoff functions $(p_l, p_{-l}) \in [0, \infty)^3 \mapsto \phi_l^{(a)}(p_l, p_{-l})$, $l = 1, 2, d$, is smooth strictly supermodular, parameterized by $a = 0, 1$. For all l, $\pi_l^{(a)}(p_l, p_{-l})$ has increasing differences in (p_l, a), and $\pi_k^{(a)}(p_k, p_{-k})$ has strictly increasing differences in (p_k, a). Since we assume that all configurations analyzed in this paper yield a unique downstream equilibrium, supermodularity theory (see Vives (1999), Theorem 2.3) tells us that this equilibrium is strictly increasing in a. Conversely, if the best response function of a firm shifts downwards, then this proof can be easily adapted to show that all equilibrium prices decrease. $\square$

A.2 Proof of Lemma 1

Proof. Assume that the alternative source supplies the input to firm d at price $\overline{m}$. In this case, the first-order condition of integrated firm $i \in \{1, 2\}$ is given by:

$$\frac{\partial \tilde{\pi}_i^{(\emptyset)}}{\partial p_i}(p, \overline{m}) = q_i + (p_i - c_i'(q_i) - m)\frac{\partial q_i}{\partial p_i} = 0.$$

Consider now that firm i decides to set upstream price $w = \overline{m}$, and assume that firm d switches to firm i to purchase its input. Then, firm i's first first-order condition becomes:

$$\frac{\partial \tilde{\pi}_i^{(i)}}{\partial p_i}(p, \overline{m}) = q_i + (p_i - c_i'(q_i) - m)\frac{\partial q_i}{\partial p_i} + (w - m)\frac{\partial q_d}{\partial p_i} = 0,$$

where the additional term is strictly positive since $w = \overline{m} > m$. Therefore, firm i's best response shifts upwards, while the best responses of the two other firms are not affected. By Lemma 7, this implies that all downstream prices increase. Since demand functions are decreasing and have a finite choke point, there exists $\hat{p} > p_i^{(\emptyset)}$, such that $q_i(\hat{p}, p_{-i}^{(i)}(w)) = q_i(p^{(\emptyset)}(w))$. Therefore,

$$\begin{aligned}
\pi_i^{(\emptyset)}(\overline{m}) &< \tilde{\pi}_i^{(\emptyset)}(\hat{p}, p_{-i}^{(i)}(w), w) \\
&< \tilde{\pi}_i^{(\emptyset)}(\hat{p}, p_{-i}^{(i)}(w), w) + (w - m)q_d(\hat{p}, p_{-i}^{(i)}(w)) \\
&= \tilde{\pi}_i^{(i)}(\hat{p}, p_{-i}^{(i)}(w), w) \\
&\leq \tilde{\pi}_i^{(i)}(p^{(i)}(w), w), \text{ by revealed preference.}
\end{aligned}$$

Therefore, if firm i sets $w = \overline{m}$, and if firm d accepts this new offer, then firm i's profit increases strictly. By continuity, this implies that, if firm i sets $w = \overline{m} - \varepsilon$, where $\varepsilon > 0$ is small enough, then, firm d accepts this new offer, and firm i's profit increases strictly. $\square$

A.3 Proof of Lemma 2

Proof. Assume that both integrated firms offer upstream price m. Then, by symmetry, they both earn profit $\pi_i^{(i)}(m)$. If an integrated firm, say j, deviates upwards, then the downstream firm still purchases the input at marginal cost from the other integrated firm. Therefore, firm j still earns $\pi_j^{(i)}(m) = \pi_i^{(i)}(m)$ $(j \neq i)$, and the deviation is not profitable.

If firm j deviates downward, then, the proof is along the line of the proof of Lemma 1. If firm j sets $w < m$, then its best response function and the one of firm d shift downwards, and all downstream prices decrease by Lemma 7. Therefore, the downward deviation yields negative upstream profits and reduces downstream prices. Following a standard revealed preference argument, this implies that firm j's profit decreases strictly, so that this deviation is not profitable. □

A.4 Proof of Lemma 3

Proof. In the proof of Lemma 2, we have shown that $\pi_i^{(i)}(w) < \pi_i^{(i)}(m)$ for $i = 1, 2$ and $w < m$, which implies that $w_m \geq m$. Moreover, taking the first derivative of $\pi_i^{(i)}(.)$ at point $w = m$, we get, using the envelope theorem,

$$\frac{d\pi_i^{(i)}}{dw}(m) = q_d(p^{(i)}(m)) + (p_i^{(i)}(m) - c - c'(q_i(p_i^{(i)}(m))))(\frac{\partial q_i}{\partial p_j}\frac{dp_j^{(i)}}{dw} + \frac{\partial q_i}{\partial p_d}\frac{dp_d^{(i)}}{dw}),$$

where $j \neq i$ in $\{1, 2\}$. The first term on the right-hand side is clearly positive. When w increases, the best-response function of firms i and d shift upwards. By Lemma 7, this implies that equilibrium downstream prices are increasing in w. From firm i's first order condition, $p_i^{(i)}(m) - c - c'(q_i(p_i^{(i)}(m))) > 0$, and therefore, the second term in the above equation is positive as well. As a result, $\frac{d\pi_i^{(i)}}{dw}(m) > 0$, and $w_m > m$. □

A.5 Proof of Proposition 1

Proof. Let $m < w \leq \overline{m}$ and $i \neq j$ in $\{1, 2\}$. To show that $p_i^{(i)}(w) > p_j^{(i)}(w) (= p_i^{(j)}(w))$, we denote by $B_i(p_j, p_d, w)$ (respectively $B_j(p_i, p_d, w)$) firm i (resp. j)'s best response when the upstream market is supplied by i at price w. The comparison between first order conditions 1 and 2 indicates that $B_i(., ., w) > B_j(., ., w)$.

Assume, by contradiction, that $p_i^{(i)}(w) \leq p_j^{(i)}(w)$. Then, by strategic complementarity,

$$\begin{aligned} p_j^{(i)}(w) &= B_j(p_i^{(i)}(w), p_d^{(i)}(w), w) \\ &\leq B_j(p_j^{(i)}(w), p_d^{(i)}(w), w) \\ &< B_i(p_j^{(i)}(w), p_d^{(i)}(w), w) \\ &= p_i^{(i)}(w), \end{aligned}$$

which is a contradiction. Therefore, $p_i^{(i)}(w) > p_j^{(i)}(w)$.

A straightforward revealed preference argument, as in the proof of Lemma A.2, shows that firm i earns a strictly lower downstream profit than j. □

A.6 Proof of Proposition 2

Proof. Assume that $\pi_i^{(i)}(w_m) \leq \pi_j^{(i)}(w_m)$ and let us show that $(w_m, +\infty)$ is an equilibrium. Clearly, given that firm j sets $+\infty$, using Lemma 1, and by definition of w_m, firm i does not want to set another price. In addition, since $\pi_i^{(i)}(w_m) \leq \pi_j^{(i)}(w_m)$, and again by definition of w_m, firm $U2 - D2$ does not want to undercut its rival.

Conversely, if $\pi_i^{(i)}(w_m) > \pi_j^{(i)}(w_m)$, then monopoly-like outcomes cannot be equilibria, since the integrated firm which does not supply the upstream market would rather undercut its rival.

To show that monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria, we first show that all other equilibria are symmetric. Notice first that $\pi_j^{(i)}(.)$ is increasing, since, by the envelope theorem,

$$\frac{d\pi_j^{(i)}}{dw} = (p_j^{(i)}(w) - m - c_j'(q_j(p^{(i)}(w))))(\frac{\partial q_j}{\partial p_i}\frac{dp_i^{(i)}}{dw} + \frac{\partial q_j}{\partial p_d}\frac{dp_d^{(i)}}{dw}).$$

The first term in the right-hand side product is strictly positive, thanks to firm j's first order condition. Besides, as w increases, the best responses of the upstream supplier and of the downstream firm shift upwards, so that, by Lemma 7, all downstream prices go up. Therefore, the second term in the product is positive as well, and $\pi_j^{(i)}(.)$ is an increasing function.

Let $w_i \neq w_m$ and $w_j > w_i$, and assume, by contradiction, that (w_i, w_j) is a pair of upstream prices that can be sustained in a subgame-perfect equilibrium. Then, $w_j \leq w_m$, otherwise the upstream supplier would rather set w_m. If $\pi_i^{(i)}(w_i) > \pi_j^{(i)}(w_i)$, then firm j has a strictly profitable deviation: setting $w_i - \varepsilon$. If $\pi_i^{(i)}(w_i) \leq \pi_j^{(i)}(w_i)$, then $\pi_i^{(i)}(w_i) < \pi_j^{(i)}(w_j)$ since $\pi_j^{(i)}(.)$ is increasing, and firm i has a strictly profitable deviation: setting $w_j + \varepsilon$. In both cases we get a contradiction.

Assume now that $\pi_i^{(i)}(w_m) < \pi_j^{(i)}(w_m)$, and consider a monopoly-like equilibrium $(w_m, +\infty)$, and another equilibrium, which we know is symmetric, (w, w). Obviously, $\pi_i^{(i)}(w) = \pi_j^{(i)}(w)$, otherwise one of the two firms would rather undercut or exit the upstream market. Besides, $w < w_m$, otherwise, the upstream supplier would prefer to set w_m. Then we have, by definition of w_m, $\pi_j^{(i)}(w) = \pi_i^{(i)}(w) < \pi_i^{(i)}(w_m) < \pi_j^{(i)}(w_m)$, which proves that monopoly-like equilibria Pareto-dominate all other equilibria.

We now show that all other equilibria than the monopoly-like equilibria involve weakly dominated strategies on the upstream market. We have just seen that these other equilibria are of the form (w, w), with $w < w_m$ and $\pi_i^{(i)}(w) = \pi_j^{(i)}(w)$. Let us show that offering $w_i = w_m$ weakly dominates offering $w_i = w$ for integrated firm i. If the integrated rival offers $w_j \leq w$, then both strategies yield the same payoffs. If $w < w_j < w_m$, then offering w_m yields a payoff $\pi_j^{(i)}(w_j)$, which is larger than the payoff when offering w, $\pi_i^{(i)}(w) = \pi_j^{(i)}(w)$, because $\pi_j^{(i)}(.)$ is increasing. If $w_j > w_m$, then offering w_m yields a payoff $\pi_i^{(i)}(w_m)$, which is larger than the payoff when offering w, $\pi_i^{(i)}(w)$, by definition of w_m. If $w_j = w_m$, the former two cases show that it is also strictly preferable to offer w_m rather than w. □

A.7 Proof of Proposition 3

Proof. Assume that integrated firm i supplies the upstream market at price w, and denote its integrated rival by j. To begin with, it is straightforward to see that we can normalize the intercepts of the linear demands to $D = 1$ and all upstream and downstream costs to $c = m = 0$, by redefining upstream prices as $\frac{a_i - c_u}{D - c - c_u}$ and downstream prices as $\frac{p_k - c - c_u}{D - c - c_u}$.

Equilibrium downstream prices are given by:

$$\begin{aligned}
p_i^{(i)}(w) &= \frac{18 + \gamma(15 + w(9 + 5\gamma))}{2(3 + \gamma)(6 + 5\gamma)}, \\
p_j^{(i)}(w) &= \frac{3(6 + \gamma(5 + w + w\gamma))}{2(3 + \gamma)(6 + 5\gamma)}, \\
p_d^{(i)}(w) &= \frac{3(6 + 5\gamma) + w(18 + 7\gamma(3 + \gamma))}{2(3 + \gamma)(6 + 5\gamma)},
\end{aligned}$$

and profits are equal to:

$$\begin{aligned}
\pi_i^{(i)}(w) &= \frac{3(3+\gamma)(6+5\gamma)^2+6w(1+\gamma)(6+5\gamma)(18+\gamma(18+5\gamma))-w^2(1+\gamma)(648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4)}{4(3+\gamma)^2(6+5\gamma)^2},\\
\pi_j^{(i)}(w) &= \frac{3(3+2\gamma)(6+\gamma(5+w+w\gamma))^2}{4(3+\gamma)^2(6+5\gamma)^2},\\
\pi_d^{(i)}(w) &= \frac{3(3+2\gamma)(6+5\gamma-w(1+\gamma)(6+\gamma))^2}{4(3+\gamma)^2(6+5\gamma)^2}.
\end{aligned}$$

$\pi_i^{(i)}(.)$ is strictly concave and reaches its maximum value for

$$w_m = \frac{3(6+5\gamma)(18+\gamma(18+5\gamma))}{648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4}.$$

π_d is strictly lower and strictly positive if it purchases the input from the alternative source at price w_m. We assume that the price of the alternative source of input does not constrain the monopoly upstream price, $\overline{m} > w_m$.

$\pi_i^{(i)}(.)$ and $\pi_j^{(i)}(.)$ are parabolas, they cross each other twice, at $w_i = m$ and at $w_i = w_*$, where

$$w_* \equiv \frac{9(12+16\gamma+5\gamma^2)}{108+180\gamma+93\gamma^2+13\gamma^3}.$$

$\pi_i^{(i)}(.)$ is strictly concave and $\pi_j^{(i)}(.)$ is convex since $\frac{d^2\pi_j^{(i)}}{dw_i^2} = \frac{3(3+2\gamma)\gamma^2(1+\gamma)^2}{2(3+\gamma)^2(6+5\gamma)^2} \geq 0$. Hence, we have:

$$\pi_i^{(i)}(w_i) \geq \pi_j^{(i)}(w_i) \quad \Leftrightarrow \quad w_i \in [0, w_*]. \tag{11}$$

Let us now check whether $w_m \in [0, w_*]$:

$$w_m - w_* = \frac{3(3+\gamma)(6+5\gamma)(-648-1296\gamma-864\gamma^2-183\gamma^3+5\gamma^4)}{(108+180\gamma+93\gamma^2+13\gamma^3)(648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4)}.$$

Analyzing the above function, we establish that there exists $\overline{\gamma} > 0$, such that $w_m \geq w_*$ if, and only if, $\gamma \geq \overline{\gamma}$.

If $\gamma \geq \overline{\gamma}$, monopoly-like outcomes are equilibria. We know from Proposition 2 that all other equilibria are symmetric. The two candidates are $w = m$ and $w = w_*$. Clearly, both are equilibrium outcomes.

If $\gamma < \overline{\gamma}$, then, there are no monopoly-like equilibria. w_* cannot be sustained in equilibrium, since $w_* > w_m$. The only candidate left is the perfect competition outcome, which is always an equilibrium. □

A.8 Proof of Proposition 4

Proof. The demand functions can be written as $q_k = 1/3 + 1/(2t)(p_{k'} + p_{k''} - 2p_k)$. As in Appendix A.7, we can normalize m and c to 0, and t to 1 without loss of generality.

Assuming that firm i supplies the upstream market at price w, and solving for the Nash equilibrium of the downstream competition subgame, we get:

$$\begin{aligned}
p_i^{(i)}(w) &= \frac{\delta}{5}+\frac{1}{3}+\frac{w}{2},\\
p_j^{(i)}(w) &= \frac{1}{30}(6\delta+10+9w),\\
p_d^{(i)}(w) &= \frac{3\delta}{5}+\frac{1}{3}+\frac{7w}{10}.
\end{aligned}$$

With these equilibrium downstream prices, it is straightforward to show that, when the input is priced at marginal cost, the downstream firm's demand lies between 0 and 1, as long as $-5/3 < \delta < 5/6$, which we assume in the following.

Profits are given by:

$$\pi_i^{(i)}(w) = \frac{1}{450}(2(5+3\delta)^2 + 45(5-3\delta)w - 135w^2),$$
$$\pi_j^{(i)}(w) = \frac{1}{900}(10+6\delta+9w)^2.$$

$\pi_i^{(i)}(.)$ and $\pi_j^{(i)}(.)$ are concave and convex parabolas, respectively. They intersect each other twice: at $w = m$ and at $w = w_* \equiv \frac{2}{13}(5-7\delta)$.

Notice first that, when $\delta \geq 5/7$, w_* is negative. Therefore, $\pi_i^{(i)}(w) \leq \pi_j^{(i)}(w)$ for all $w > 0$, and, in particular, monopoly-like equilibria exist.

Assume now that $\delta < 5/7$. It is easy to see that $w = w_*$ always allows the downstream firm to be active on the downstream market. Besides, $\pi_i^{(i)}(w) \geq \pi_j^{(i)}(w)$ if, and only if, $w \in [0, w_*]$. Now, we assume that $\overline{m}$ is not too low, namely, $w_* < \overline{m}$.

$\pi_i^{(i)}(.)$ reaches its maximum at $w = \tilde{w} \equiv \frac{1}{6}(5-3\delta)$. If $\delta \in [-1/9, 5/7]$, then, $\tilde{w} \geq w_*$. Since $w_m = \min\{\tilde{w}, \overline{m}\}$, this implies that $w_m \geq w_*$, and therefore, $\pi_i^{(i)}(w_m) \leq \pi_j^{(i)}(w_m)$: monopoly-like equilibria exist.

Conversely, if $\delta < -1/9$, $w_m = \tilde{w} < w_*$. Therefore, $\pi_i^{(i)}(w_m) > \pi_j^{(i)}(w_m)$, and there are no monopoly-like equilibria. According to Proposition 2, the remaining candidate for a partial foreclosure equilibrium is w_*. However, $w_* > w_m$. As a result, there are no partial foreclosure equilibria. □

A.9 Proof of Proposition 6

Proof. Consider a partial foreclosure equilibrium with upstream price $\hat{w} > m$. We know from the proof of Proposition 2 that all prices go up as w increases. In particular, downstream prices are strictly higher in a partial foreclosure equilibrium than in the perfect competition outcome.

Assume also: a representative consumer with a quasi-linear, continuously differentiable and quasi-concave utility function exists; firms have symmetric and identical demands; firms have the same convex downstream costs functions. Let us show that partial foreclosure lowers the social welfare.

When the upstream price is set at marginal cost, the three firms are perfectly identical. Hence, since the downstream equilibrium is unique, it is symmetric, and $p_k^{(i)}(m) = \hat{p}$ for all k. Let $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$ a permutation of the triple $\left(p_k^{(i)}(\hat{w})\right)_{k=1,2,d}$ such that $\hat{p}_1 \leq \hat{p}_2 \leq \hat{p}_3$, and let us relabel firms so that firm k is the firm that charges $\hat{p}_k$ when the upstream price is $\hat{w}$. Recall that $\hat{p}_k > \hat{p}$ for all k.

Keeping in mind that firms have been relabeled, let us denote by $U(q_0, q_1, q_2, q_3) = q_0 + u(q_1, q_2, q_3)$ the utility function of the representative consumer, where q_0 denotes consumption of the numeraire, and q_k denotes consumption of product $k \in \{1, 2, 3\}$. We can then write the social welfare as a function of the downstream price vector p:[50]

$$W(p) = u\left(q_1(p), q_2(p), q_3(p)\right) - \sum_{k=1}^{3} \left\{m q_k(p) + c\left(q_k(p)\right)\right\},$$

where $c(.)$ denotes the downstream cost function, which, by assumption, is the same for the three firms. This welfare function is continuously differentiable, since functions u, c and D_k are continuously differentiable. To prove the result, we need to show that $W(\hat{p}_1, \hat{p}_2, \hat{p}_3) - W(\hat{p}, \hat{p}, \hat{p})$, the variation in social welfare when

[50] This function does not depend on the upstream price.

downstream prices increase from $(\hat{p},\hat{p},\hat{p})$ to $(\hat{p}_1,\hat{p}_2,\hat{p}_3)$, is strictly negative. This variation can be written as:

$$\begin{aligned}(W(\hat{p}_1,\hat{p}_2,\hat{p}_3)-W(\hat{p}_1,\hat{p}_2,\hat{p}_2))+(W(\hat{p}_1,\hat{p}_2,\hat{p}_2)-W(\hat{p}_1,\hat{p}_1,\hat{p}_1))+(W(\hat{p}_1,\hat{p}_1,\hat{p}_1)-W(\hat{p},\hat{p},\hat{p}))\\ =\int_{\hat{p}_2}^{\hat{p}_3}\frac{\partial W}{\partial p_3}(\hat{p}_1,\hat{p}_2,r)dr+\int_{\hat{p}_1}^{\hat{p}_2}\sum_{k=2}^{3}\frac{\partial W}{\partial p_k}(\hat{p}_1,r,r)dr+\int_{\hat{p}}^{\hat{p}_1}\sum_{k=1}^{3}\frac{\partial W}{\partial p_k}(r,r,r)dr.\end{aligned}$$

We know from Proposition 1 that $\hat{p}_1<\hat{p}_2$ or $\hat{p}_2<\hat{p}_3$. Assume first that $\hat{p}_2<\hat{p}_3$. Then, we claim that the first integral in the right-hand side is strictly negative, while the two other ones are non-positive. Let us start with the first one. Let $r\in(\hat{p}_2,\hat{p}_3]$. Then,

$$\begin{aligned}\frac{\partial W}{\partial p_3}(\hat{p}_1,\hat{p}_2,r) &= \sum_{k=1}^{3}\left(\frac{\partial u}{\partial q_k}\frac{\partial q_k}{\partial p_3}-(c_u+c'(q_k))\frac{\partial q_k}{\partial p_3}\right),\\ &= \frac{\partial q_3}{\partial p_3}\left(r-c_u-c'(q_3)\right)+\sum_{k=1}^{2}\frac{\partial q_k}{\partial p_3}\left(\hat{p}_k-c_u-c'(q_k)\right).\end{aligned}$$

Let $k\in\{1,2\}$. Firm 3 has the highest markup: $r-c_u-c'(q_3)>\hat{p}_k-c_u-c'(q_k)$. Indeed, since $r>\hat{p}_k$, $q_3<q_k$, and, since costs are convex, $c'(q_3)<c'(q_k)$. Besides, firm 3's markup is strictly positive. Indeed, since $q_3(\hat{p}_1,\hat{p}_2,r)<\frac{1}{3}\sum_{k=1}^{3}q_k(\hat{p}_1,\hat{p}_2,r)\le\frac{1}{3}\sum_{k=1}^{3}q_k(\hat{p},\hat{p},\hat{p})=q_3(\hat{p},\hat{p},\hat{p})$, then $r-c_u-c'(q_3(\hat{p}_1,\hat{p}_2,r))>\hat{p}-c_u-c'(q_3(\hat{p},\hat{p},\hat{p}))$, which is strictly positive, since $\pi(c_u)>0$. As a result,

$$\frac{\partial W}{\partial p_3}(\hat{p}_1,\hat{p}_2,r)<(r-c_u-c'(q_3))\sum_{k=1}^{3}\frac{\partial q_k}{\partial p_3}<0.$$

Therefore, the first integral is indeed strictly negative, since $\hat{p}_2<\hat{p}_3$. A similar argument shows that the two other integrands are non-positive, and we can conclude that the social welfare is strictly lower in a partial foreclosure equilibrium.

If $\hat{p}_1<\hat{p}_2$, we can make the same reasoning to show that the first and third integrals are non-positive, while the second one is strictly negative. This concludes the proof. □

A.10 Proof of Proposition 7

Proof. Let $i\neq j$ in $\{1,2\}$. We show that the conditions stated in Proposition 7 are sufficient to have $\frac{d\pi_i^{(i)}}{dw_i}(m)>\frac{d\pi_j^{(i)}}{dw_i}(m)$. To simplify the exposition, we introduce the following notations. p denotes the equilibrium downstream price set by the three firms when the upstream market is supplied at marginal cost. q denotes the demand addressed to each firm when all downstream prices are equal to p. We also denote by c' and c'' the first and second derivatives of the downstream cost function when the quantity produced is q. Last, we define $\delta\equiv\frac{\partial q_k}{\partial p_k}$, $\tilde{\delta}\equiv\frac{\partial q_k}{\partial p_{k'}}$, $\gamma\equiv\frac{\partial^2 q_k}{\partial p_k^2}$ and $\tilde{\gamma}\equiv\frac{\partial^2 q_k}{\partial p_k\partial p_{k'}}$, where all the derivatives are taken at price vector (p,p,p), and $k\neq k'$ in $\{1,2,d\}$.[51]

With these notations, when $w_i=m$, the first-order conditions on the downstream market can be rewritten as $(p-m-c')\delta+q=0$. The second-order conditions are given by $(2-\delta c'')\delta+(p-m-c')\gamma<0$. Since downstream prices are strategic complements, $(1-\delta c'')\tilde{\delta}+(p-m-c')\tilde{\gamma}\ge 0$. Besides, since the total demand is decreasing, $\delta+2\delta'\le 0$.

[51]Notice that, since the three firms are identical, p, q, c', c'', δ, $\tilde{\delta}$, γ and $\tilde{\gamma}$ are well-defined, and do not depend on k or k'.

Differentiating the profit functions with respect to w_i for $w_i = m$, we obtain:

$$\begin{aligned}
\frac{d\pi_i^{(i)}}{dw_i}(m) &= (p-m-c')\tilde{\delta}\left(\frac{dp_j^{(i)}}{dw_i}(m) + \frac{dp_d^{(i)}}{dw_i}(m)\right) + q, \\
\frac{d\pi_j^{(i)}}{dw_i}(m) &= (p-m-c')\tilde{\delta}\left(\frac{dp_i^{(i)}}{dw_i}(m) + \frac{dp_d^{(i)}}{dw_i}(m)\right).
\end{aligned}$$

Therefore,

$$\frac{d\pi_i^{(i)}}{dw_i}(m) - \frac{d\pi_j^{(i)}}{dw_i}(m) = q - (p-m-c')\tilde{\delta}\left(\frac{dp_i^{(i)}}{dw_i}(m) - \frac{dp_j^{(i)}}{dw_i}(m)\right). \tag{12}$$

As usual, we obtain the expression of $\frac{dp_i^{(i)}}{dw_i}(m) - \frac{dp_j^{(i)}}{dw_i}(m)$ by differentiating firms i and j's first-order conditions with respect to w_i. We get:

$$\frac{dp_i^{(i)}}{dw_i}(m) - \frac{dp_j^{(i)}}{dw_i}(m) = \frac{\tilde{\delta}}{-\Big((2-\delta c'')\delta + (p-m-c')\gamma - ((1-\delta c'')\tilde{\delta} + (p-m-c')\tilde{\gamma})\Big)}.$$

Plugging this into equation (12), using the first-order conditions to get rid of the q term, and rearranging terms, we finally obtain:

$$\begin{aligned}
\frac{d\pi_i^{(i)}}{dw_i}(m) - \frac{d\pi_j^{(i)}}{dw_i}(m) = {} & \frac{p-m-c'}{-\Big((2-\delta c'')\delta + (p-m-c')\gamma - ((1-\delta c'')\tilde{\delta} + (p-m-c')\tilde{\gamma})\Big)} \\
& \times \Big(-\tilde{\delta}^2 + \delta\Big((2-\delta c'')\delta + (p-m-c')\gamma - ((1-\delta c'')\tilde{\delta} + (p-m-c')\tilde{\gamma})\Big)\Big).
\end{aligned}$$

Strategic complementarity and second-order conditions imply that the denominator in the right-hand side is positive. Therefore, the above expression is positive if, and only if

$$-\tilde{\delta}^2 + (-\delta)\Big((-\delta)(2-\delta c'') - (p-m-c')\gamma + \tilde{\delta}(1-\delta c'') + (p-m-c')\tilde{\gamma}\Big) > 0.$$

Since $\delta < 0$, $-\delta > \tilde{\delta} > 0$, $p-m-c' > 0$, $c'' > 0$, $(-\delta)(2-\delta c'') - (p-m-c')\gamma > 0$, and $\tilde{\delta}(1-\delta c'') + (p-m-c')\tilde{\gamma} \geq 0$, this is the case if:

- $\gamma \leq 0$,
- or $\tilde{\gamma} \geq 0$.

□

A.11 Proof of Proposition 8

As usual, ne normalize m and c to 0, and t to 1 without loss of generality.

We first prove that, with our linear demands, any repartition of the input demand among the cheapest upstream suppliers can be supported by equilibrium strategies. We denote by $S_I \equiv \{1, 2, \ldots M\}$ (resp. $S_D \equiv \{d_1, d_2, \ldots d_N\}$) the sets of integrated (resp. unintegrated downstream) firms. $S \equiv S_I \cup S_D$ is the set of all firms. For all $(i,k) \in S_I \times S_D$, we denote by $\alpha_{ik} \in [0,1]$ the proportion of input that downstream firm k purchases from integrated firm i. We prove the following lemma:

Lemma 8. *Let $\alpha = (\alpha_{ik})_{(i,k)\in S_I\times S_D} \in [0,1]^{MN}$, such that $\sum_{i\in S_I}\alpha_{ik} = 1$ for all $k \in S_D$, a repartition of the upstream demand. Then, α can be sustained in a subgame-perfect equilibrium if, and only if, for all $(i,k) \in S_I \times S_D$, $\alpha_{ik} = 0$ whenever $w_i > \min_{j\in S_I} w_j$.*

Proof. Assume that α is an equilibrium repartition. Since downstream firms can switch to another supplier at zero cost, once downstream prices have been set, a firm i whose upstream price w_i is strictly larger than $\min_{j\in S_I} w_j$ does not sell any input on the upstream market: if $w_i > \min_{j\in S_I} w_j$, then $\alpha_{ik} = 0$ for all $k \in S_D$.

Conversely, let $\alpha = (\alpha_{ik})_{(i,k)\in S_I\times S_D} \in [0,1]^{MN}$, such that $\sum_{i\in S_I}\alpha_{ik} = 1$ for all $k \in S_D$, and $\alpha_{ik} = 0$ whenever $w_i > \min_{j\in S_I} w_j$. In the following, we prove that the profit earned by any firm $k \in S_D$ does not depend on α.

For a given α, firms' payoffs at the downstream competition stage are given by:

$$\begin{aligned}\tilde{\pi}_i &= p_i q_i + w \sum_{k\in S_D} \alpha_{ik} q_k, \text{ for all } i \in S_I,\\ \tilde{\pi}_k &= (p_k - w - \delta) q_k, \text{ for all } k \in S_N.\end{aligned}$$

The first-order condition for firm $i \in S_I$ is given by:

$$\frac{\partial \tilde{\pi}_i}{\partial p_i} = 0 = q_i + p_i \frac{\partial q_i}{\partial p_i} + w \sum_{k\in S_D} \alpha_{ik} \frac{\partial q_k}{\partial p_i}.$$

The second-order condition holds. Using equation (6) and rearranging terms, we get:

$$\frac{2(M+N)-1}{2} p_i = \frac{1}{M+N} + \frac{1}{2}\left(\sum_{k\in S} p_k + w \sum_{k\in S_D} \alpha_{ik}\right). \tag{13}$$

Taking the difference between firm i's and firm 1's first-order condition, we get:

$$p_i = p_1 + \frac{w}{2(M+N)-1} \sum_{k\in S_D} (\alpha_{ik} - \alpha_{1k}). \tag{14}$$

Using a similar argument, we show that

$$p_k \frac{2(M+N)-1}{2} = \frac{1}{M+N} + \frac{1}{2}\left(\sum_{k'\in S} p_{k'} + (w+\delta)(M+N-1)\right), \tag{15}$$

which implies in particular that $p_k = p_{k'} = \hat{p}$ for all $k, k' \in S_D$. Taking the difference between equations (13) and (15), we get:

$$\hat{p} = p_1 + \frac{1}{2(M+N)-1}\left((w+\delta)(M+N-1) - w \sum_{k\in S_D} \alpha_{1k}\right). \tag{16}$$

Plugging equations (14) and (16) into equation (13), using the fact that $\sum_{i\in S_I}\alpha_{ik} = 1$ for all k, and rearranging terms, we obtain

$$p_1 = \frac{2}{(M+N)(M+N-1)} + \frac{1}{2(M+N)-1}\left(N(w+\delta) + w\Big(\frac{N}{M+N-1} + \sum_{k\in S_N} \alpha_{1k}\Big)\right),$$

which gives us the downstream equilibrium prices of all firms:

$$p_i = \frac{2}{(M+N)(M+N-1)} + \frac{1}{2(M+N)-1}\left(N(w+\delta) + w(\frac{N}{M+N-1} + \sum_{k\in S_N} \alpha_{ik})\right), \tag{17}$$

$$p_k = \frac{2}{(M+N)(M+N-1)} + \frac{1}{2(M+N)-1}\left((w+\delta)(M+2N-1) + w\frac{N}{M+N-1}\right), \tag{18}$$

for all $i \in S_I$, $k \in S_D$.

Notice that the equilibrium prices set by downstream firms do not depend of α. Plugging expressions (17) and (18) into the demand function, we get:

$$q_k = \frac{1}{M+N} + \frac{1}{2}\frac{wN - (w+\delta)M(M+N-1)}{2(M+N)-1}, \tag{19}$$

for all $k \in S_D$. Therefore downstream firms' equilibrium prices and quantities do not depend on α. This implies that their profits do not depend on α either, so that downstream firms are indifferent between any repartition α that satisfies $\alpha_{ik} = 0$ whenever $w_i > \min_{j\in S_I} w_j$. All these repartitions can therefore be sustained in a subgame-perfect equilibrium. □

We can now prove Proposition 8:

Proof. Assume that integrated firm 1 supplies the upstream market at the monopoly upstream price, and let us see whether firm 2 wants to undercut. The downstream equilibrium prices are given by equations (17) and (18), with, for all $k \in S_D$, $\alpha_{1k} = 1$, and $\alpha_{ik} = 0$ if $i \geq 2$.

Notice, from equation (19), that q_k, the demand addressed to firm $k \in S_D$ at the downstream equilibrium, is strictly decreasing in δ. Therefore, if $w = 0$, there exists $\bar{\delta} > 0$, such that $q_k > 0$ if, and only if, $\delta < \bar{\delta}$. This maximum value of δ is equal to:

$$\bar{\delta} = \frac{2(\frac{1}{M+N} + \frac{1}{M+N-1})}{M}.$$

We assume in the following that all downstream firms can be active when the upstream price is set at marginal cost: $\delta \geq \bar{\delta}$. The profit of integrated firm 1 at the subgame equilibrium is given by:

$$\begin{aligned}
\pi_1 = \big(&4 - 16M + 16M^2 - 16N + 32MN + 4\delta MN - 12\delta M^2N + 8\delta M^3N + 16N^2 + 4\delta N^2 - 24\delta MN^2 \\
&+ 24\delta M^2N^2 + \delta^2M^2N^2 - 2\delta^2M^3N^2 + \delta^2M^4N^2 - 12\delta N^3 + 24\delta MN^3 + 2\delta^2MN^3 - 6\delta^2M^2N^3 \\
&+ 4\delta^2M^3N^3 + 8\delta N^4 + \delta^2N^4 - 6\delta^2MN^4 + 6\delta^2M^2N^4 - 2\delta^2N^5 + 4\delta^2MN^5 + \delta^2N^6 - N(M+N)^2 \\
&\times(-1+2M+2N)(2 + (-4+\delta(-1+M)^2)M - 4N + \delta(-1+M)(-1+2M)N + \delta(-1+M)N^2)w \\
&-(-1+M)N(-1+M+N)(M+N)^3(-1+2M+2N)w^2\big) \,/\, \big(2(-1+M+N)(M+N)^2(-1+2M+2N)^2\big).
\end{aligned}$$

The profit of integrated firm 2 can be written as:

$$\pi_2 = \frac{(-2 + M^2N(\delta+w) + N(4 + \delta(-1+N)N + N^2w) + M(4 + \delta N(-1+2N) + 2N^2w))^2}{(2(-1+M+N)(M+N)^2(-1+2M+2N)^2)}$$

Since

$$\frac{\partial^2(\pi_2 - \pi_1)}{\partial w^2} = \frac{N(M+N)(\frac{N(M+N)}{-1+M+N} + (-1+M)(-1+2M+2N))}{(-1+2M+2N)^2} > 0,$$

$\pi_2 - \pi_1$ is convex in w, and hence, positive outside its roots. The two roots are $w = 0$, and $w = w_*$, where

$$w_* = -((2M-1)(6+(-4+\delta(M-1)^2)M) + (M-1)(-16+\delta-4\delta M+6\delta M^2)N + (-8+\delta(1+M(6M-5)))N^2 + 2\delta MN^3)/((M+N)(-1+2M^3+2M(N-2)(N-1)-(N-3)N+M^2(4N-5))).$$

The derivative of w_* with respect to δ is equal to:

$$\frac{\partial w_*}{\partial \delta} - \frac{(M+N-1)(1+M(2M+2N-3))}{-1+2M^3+2M(N-2)(N-1)-(N-3)N+M^2(4N-5)},$$

which is strictly negative for all $M \geq 2$, $N \geq 1$. Therefore, w_* is positive, if, and only if, $\delta < \delta_*$, where

$$\delta_* \equiv \frac{2(2M+2N-3)(-1+2M+2N)}{(M+N-1)(M+N)(1+M(2M+2N-3))}.$$

Notice that $\bar{\delta} > \delta_*$, since

$$\bar{\delta} - \delta_* = \frac{4M+4N-2}{M(M+N-1)(M+N)(1+M(2M+2N-3))} > 0.$$

This implies that, when $\delta \in [\delta_*, \bar{\delta})$, for all $0 < w < \overline{m}$, $\pi_1(w) < \pi_2(w)$. In particular, $\pi_1(w_m) < \pi_2(w_m)$, and monopoly-like equilibria exist when $\delta \in [\delta_*, \bar{\delta})$.

Assume now that $\delta < \delta_*$, i.e., $w_* > 0$. We first show that, for all $\delta < \delta_*$, downstream firms are active when $w = w_*$. For all $k \in S_D$, the derivative of $q_k(w_*)$ with respect to δ is given by:

$$\frac{\partial q_k(w_*)}{\partial \delta} = -\frac{N(M+N-1)(1+M(M+N-1))}{2(2M+2N-1)(-1+2M^3+2M(N-2)(N-1)-(N-3)N+M^2(4N-5))},$$

which is negative. We know that, when $\delta = \delta_*$, $w_* = 0$, and therefore, $q_k(w_*) = q_k(0) > 0$, since $\delta_* < \bar{\delta}$. As a result, $q_k(w_*) > 0$ for all $\delta < \delta_*$. Now the question is whether $w_m \lesseqgtr w_*$.

Let us compute $\tilde{w} \equiv \arg\max_w \pi_1(w)$. $\tilde{w}$ is well-defined, since $\pi_1(.)$ is strictly concave. Using the first-order condition, we get:

$$\tilde{w} = -\frac{\delta}{2} + \frac{\frac{1}{M+N-1} + \frac{1}{M+N}}{M-1}.$$

It is then straightforward to show that $\tilde{w} > w_*$ if, and only if, $\delta > \delta^m(M,N)$, where

$$\delta^m(M,N) \equiv \frac{2(2M+2N-1)((M-1)^2(2M-5)+(7+4(M-3)M)N+(2M-3)N^2)}{(M-1)(M+N-1)(M+N)(-1+2M^3+(N-1)N+M^2(4N-5)+2M(2+(N-2)N))}. \tag{20}$$

$\delta_* - \delta^m(M,N)$ is equal to:

$$\frac{4(2M+2N-1)(-1+2M^3+2M(N-2)(N-1)-(N-3)N+M^2(4N-5))}{(M-1)(M+N)(1+M(2M+2N-3))(-1+2M^3+(N-1)N+M^2(4N-5)+2M(2+(N-2)N))},$$

which is positive. Consider first that $\delta < \delta^m(M,N)$. In this case, $w_m = \min\{w_m, \overline{m}\} < w_*$, and therefore, there are no monopoly-like equilibria. By contrast, if $\delta^m(M,N) \leq \delta \leq \delta_*$, then, as long as $\overline{m}$ is not too small, $w_m > w_*$, and monopoly-like equilibria exist.

We can conclude that monopoly-like equilibria exist if, and only if, $\delta \geq \delta^m(M,N)$. □

A.12 Proof of Proposition 9

Proof. **Behavior of δ^m as a function of M:** Computing the corresponding derivative, we get:

$$\frac{\partial \delta^m}{\partial M} = \frac{\sum_{k=0}^{7} P_k(M)N^k}{(M-1)^2(M+N-1)^2(M+N)^2(-1+2M^3-N+N^2+M^2(4N-5)+2M(2-2N+N^2))^2},$$

where

$$
\begin{aligned}
P_0(M) &= 10-120M+618M^2-1792M^3+3222M^4-3720M^5+2750M^6-1248M^7+312M^8-32M^9,\\
P_1(M) &= -50+696M-3424M^2+8704M^3-12962M^4+11720M^5-6284M^6+1808M^7-208M^8,\\
P_2(M) &= 188-2112M+8518M^2-17408M^3+20030M^4-13048M^5+4408M^6-576M^7,\\
P_3(M) &= -430+3592M-11200M^2+17408M^3-14332M^4+5840M^5-880M^6,\\
P_4(M) &= 556-3424M+7994M^2-8848M^3+4520M^4-800M^5,\\
P_5(M) &= -390+1784M-2980M^2+2032M^3-432M^4,\\
P_6(M) &= 138-472M+488M^2-128M^3,\\
P_7(M) &= -20+48M-16M^2.
\end{aligned}
$$

All the above polynomials are negative for M high enough. Their largest roots are (approximately) 3.3802, 3.23673, 3.08447, 2.92684, 2.76987, 2.6256, 2.51816 and 2.5 respectively. This implies that all these polynomials are strictly negative whenever $M \geq 4$. Therefore, $\partial\delta^m/\partial M < 0$ when $M \geq 4$. To conclude the analysis, we need to sign the impact on δ^m when M increases from 2 to 3, and from 3 to 4:

$$
\begin{aligned}
\delta^m(4,N)-\delta^m(3,N) &= -\frac{N(5420+10437N+7572N^2+2576N^3+403N^4+22N^5)}{3(N+3)(40+66N+37N^2+7N^3)(252+251N+83N^2+9N^3)} < 0,\\
\delta^m(3,N)-\delta^m(2,N) &= \frac{420+1463N+1791N^2+988N^3+249N^4+27N^5+2N^6}{(N+2)(3+10N+12N^2+5N^3)(60+89N+44N^2+7N^3)} > 0.
\end{aligned}
$$

Behavior of δ^m as a function of N: Computing the corresponding derivative, we get:

$$
\frac{\partial\delta^m}{\partial N} = \frac{\sum_{k=0}^{6} P_k(M)N^k}{(M-1)(M+N-1)^2(M+N)^2(-1+2M^3-N+N^2+M^2(4N-5)+2M(2-2N+N^2))^2},
$$

where

$$
\begin{aligned}
P_0(M) &= -10+60M-176M^2+376M^3-602M^4+636M^5-396M^6+128M^7-16M^8,\\
P_1(M) &= -32M+332M^2-1184M^3+1964M^4-1640M^5+656M^6-96M^7,\\
P_2(M) &= 94-244M-392M^2+2008M^3-2588M^4+1360M^5-240M^6,\\
P_3(M) &= -200+360M+600M^2-1872M^3+1440M^4-320M^5,\\
P_4(M) &= 170-148M-548M^2+800M^3-240M^4,\\
P_5(M) &= -68-8M+208M^2-96M^3,\\
P_6(M) &= 12+16M-16M^2.
\end{aligned}
$$

All the above polynomials are negative for M high enough. Their largest roots are (approximately) 3.16256, 2.9839, 2.77883, 2.54293, 2.26688, 1.93426 and 1.5. This implies that all these polynomials are strictly negative whenever $M \geq 4$. Therefore, $\partial\delta^m/\partial N < 0$ when $M \geq 4$. Now, let us sign the impact on δ^m of an increase in N, when $M = 2$ or 3:

$$
\delta^m(3,N+1)-\delta^m(3,N) = -\frac{(2(40+666N+1127N^2+723N^3+203N^4+21N^5))}{(3+N)(40+66N+37N^2+7N^3)(200+198N+65N^2+7N^3)},
$$

which is negative. Therefore, $\delta^m(3,.)$ is decreasing.

$$
\delta^m(2,N+1)-\delta^m(2,N) = \frac{-4(-60-191N-186N^2-53N^3+10N^4+5N^5)}{(2+N)(3+10N+12N^2+5N^3)(45+66N+32N^2+5N^3)}.
$$

The denominator of the above expression is positive. Its numerator is negative for N high enough. Its largest root is 3.88848. Therefore, $\delta^m(3,N)$ increases as N goes from 1 to 4, and decreases afterwards.
Behavior of $\delta^m(M, L-M)$ as a function of M: Computing the corresponding derivative (and replacing $L-M$ by N afterwards), we get:

$$\frac{\partial \delta^m(M,L-M)}{\partial M} = \frac{\sum_{k=0}^{5} P_k(M) N^k}{(M-1)^2(M+N-1)(M+N)(-1+2M^3+(N-1)N+M^2(4N-5)+2M(2+(N-2)N))^2},$$

where

$$\begin{aligned}
P_0(M) &= 50 - 332M + 908M^2 - 1336M^3 + 1146M^4 - 572M^5 + 152M^6 - 16M^7,\\
P_1(M) &= -232 + 1256M - 2780M^2 + 3216M^3 - 2036M^4 + 656M^5 - 80M^6,\\
P_2(M) &= 398 - 1772M + 3124M^2 - 2696M^3 + 1104M^4 - 160M^5,\\
P_3(M) &= -328 + 1184M - 1592M^2 + 896M^3 - 160M^4,\\
P_4(M) &= 130 - 380M + 344M^2 - 80M^3,\\
P_5(M) &= -20 + 48M - 16M^2.
\end{aligned}$$

Again, all the above polynomials are negative for M high enough. Their largest roots are (approximately) 3.67227, 3.49444, 3.29882, 3.07841, 2.8199, 2.5. This implies that all these polynomials are strictly negative whenever $M \geq 4$. Therefore, $\delta^m(M, L-M)$ increases when M increases, for $M \geq 4$. Now, let us see what happens when $M = 2$ or 3. If $N \geq 2$, then,

$$\delta^m(3,N) - \delta^m(4,N-1) = \frac{(5+2N)(-60+N(-61+N(18+N(38+11N))))}{3(2+N)(3+N)(20+N(23+7N))(25+N(29+9N))}.$$

The denominator of the above expression is positive. Its numerator is positive as well for N high enough. Its largest root is 1.27761. Therefore, $\delta^m(3,N) - \delta^m(4,N-1)$ is positive for all $N \geq 2$.

When $M = 2$,

$$\delta^m(2,N) - \delta^m(3,N-1) = -\frac{(3+2N)(8+N(29+N(2+N)(20+N)))}{(1+N)(2+N)(3+N(7+5N))(4+N(9+7N))} < 0.$$

This concludes the proof. □

A.13 Proof of Proposition 10

Proof. Assume that firm 1 offers $w = p^m - c_1 + \varepsilon$, where $\varepsilon > 0$ can be made arbitrarily small.

Assume, by contradiction, that downstream equilibrium prices are such that firm d is not active at the subgame equilibrium: $q_d(p_1^{(1)}(w), p_d^{(1)}(w)) = 0$. In this case, firm 1's profit is equal to:

$$\pi_1^{(1)}(w) = (p_1^{(1)}(w) - m - c_1) q_1(p_1^{(1)}(w), p_d^{(1)}(w)).$$

Assume, by contradiction, that $p_1^{(1)}(w) \neq p^m$. Then, since firm d is not active, and by definition of p^m,

$$\begin{aligned}
\pi_1^{(1)}(w) &< (p^m - m - c_1) q_1(p^m, +\infty)\\
&\leq (p^m - m - c_1)\left(q_1(p^m, p_d^{(1)}(w)) + q_d(p^m, p_d^{(1)}(w))\right) + \varepsilon q_d(p^m, p_d^{(1)}(w))\\
&= \tilde{\pi}_1^{(1)}(p^m, p_d^{(1)}(w), w),
\end{aligned}$$

where the second inequality comes from the fact that the total demand is non-increasing in prices. This implies that firm 1 would have a strictly profitable deviation: set p^m. This is a contradiction.

Therefore, $p_1^{(1)}(w) = p^m$. But in this case, firm d can set $\hat{p}_d = p^m + c_d - c_1 + \varepsilon + \eta$, with $\eta > 0$. Given condition (7), we know that $q_d(p^m, \hat{p}_d) > 0$ for ε and η small enough. Therefore, setting $\hat{p}_d$ enables firm d to make strictly positive profits: a contradiction. Therefore, firm d is active at the subgame equilibrium.

Since prices are strategic complements, and since $p_d^{(1)}(w) < \infty$,

$$\begin{aligned} p^m &= \arg\max_{p_1} \{(p_1 - m - c_1) q_1(p_1, +\infty) + (w - m) q_d(p_1, +\infty)\} \\ &\geq \arg\max_{p_1} \{(p_1 - m - c_1) q_1(p_1, p_d^{(1)}(w)) + (w - m) q_d(p_1, p_d^{(1)}(w))\} \\ &= p_1^{(1)}(w). \end{aligned}$$

This implies that, if firm 1 offers $p_1 = p^m$ instead of $p_1 = p_1^{(1)}(w)$, firm d still makes positive profits. Therefore,

$$\begin{aligned} \pi_1^{(1)}(w) &\geq \tilde{\pi}_1(p^m, p_d^{(1)}(w), w) \\ &= (p^m - m - c_1)\Big(q_1(p^m, p_d^{(1)}(w)) + q_d(p^m, p_d^{(1)}(w))\Big) + \varepsilon q_d(p^m, p_d^{(1)}(w)) \\ &> (p^m - m - c_1) q_1(p^m, +\infty), \end{aligned}$$

since the total demand is non-increasing, and $q_d(p^m, p_d^{(1)}(w))$ and ε are strictly positive. Therefore, complete foreclosure does not arise in equilibrium under condition (7). □

A.14 Proof of Lemma 4

Proof. Consider a demand system that satisfies properties 1, 2, 3 and 4. Let us start from the duopoly demands. Without loss of generality, they can be written as $q_i^D = 1 - p_i + \sigma p_j$.[52] Products are substitutes and the total demand is decreasing, provided that $0 < \sigma < 1$.

Now, assume that firm i is the upstream supplier, and denote by j its integrated rival. Since demands are derived from a representative consumer, they have to satisfy $\partial q_k^{(i)} / \partial p_{k'} = \partial q_{k'}^{(i)} / \partial p_k,\ k, k' \in \{1, 2, d\}$. Therefore, the triopoly demand functions can be written as follows:

$$\begin{aligned} q_i^{(i)} &= \alpha_i - \beta_i p_i + \gamma_{ij} p_j + \gamma_{id} p_d, \\ q_j^{(i)} &= \alpha_j - \beta_j p_j + \gamma_{ij} p_i + \gamma_{jd} p_d, \\ q_d^{(i)} &= \alpha_d - \beta_d p_d + \gamma_{id} p_i + \gamma_{jd} p_j. \end{aligned}$$

Solving for $q_d^{(i)} = 0$, we get $p_d = (\alpha_d + \gamma_{id} p_i + \gamma_{jd} p_j)/\beta_d$. Plugging this value of p_d into firms i and j's triopoly demand functions, and equating the resulting expressions with duopoly demands, we obtain that the α, β and γ coefficients have to satisfy the following conditions, for duopoly to be the limit case of triopoly:

$$\left\{ \begin{array}{lllll} 1 &=& \alpha_i + \frac{\gamma_{id}\alpha_d}{\beta_d} &=& \alpha_j + \frac{\gamma_{jd}\alpha_d}{\beta_d}, \\ 1 &=& \beta_i - \frac{\gamma_{id}^2}{\beta_d} &=& \beta_j - \frac{\gamma_{jd}^2}{\beta_d}, \\ \sigma &=& \gamma_{ij} + \frac{\gamma_{id}\gamma_{jd}}{\beta_d}. && \end{array} \right.$$

[52] The intercept and the own-price sensitivity effect parameters can be set to 1 up to a renormalization of prices and quantities.

Plugging these equalities into the triopoly demands of firms i and j, we get:

$$\begin{aligned} q_i^{(i)} &= 1 - p_i + \sigma p_j - \frac{\gamma_{id}}{\beta_d} q_d^{(i)}, \\ q_j^{(i)} &= 1 - p_j + \sigma p_i - \frac{\gamma_{jd}}{\beta_d} q_d^{(i)}. \end{aligned}$$

Defining $\alpha \equiv \alpha_d$, $\beta \equiv \beta_d$, $\phi \equiv \frac{\gamma_{id}+\gamma_{jd}}{2}$ and $x \equiv \frac{\gamma_{id}-\gamma_{jd}}{\gamma_{id}+\gamma_{jd}}$, we finally obtain the demand functions of the statement of Lemma 4. Firms 1, 2 and d receive a positive demand when they set a price equal to 0 if, and only if, $0 < \alpha\phi(1+|x|)/\beta < 1$. Products are substitutes, and firm d's demand decreases in its own price, if, and only if, $\phi > 0$, $-1 < x < 1$, $\beta > 0$ and $\sigma > \phi^2(1-x^2)/\beta$. Computing the first derivatives of the total demand with respect to p_i, p_j and p_d, it is easy to check that they are indeed negative if

$$0 < \phi(1+|x|)(1 - \frac{2\phi}{\beta}) < 1 - \sigma.$$

The demand functions $q_k^{(j)}$, $k = 1, 2, d$, can then be computed thanks to the *ex ante* symmetry assumption (property 3). Now, let us show that the demand system $(q_i^D)_{i=1,2}$, $(q_k^{(i)})_{i=1,2,k=1,2,d}$ can be derived from a representative consumer. Define

$$U(q_0, q_1, q_2, q_d^1, q_d^2) \equiv q_0 + Q^T A + Q^T B Q,$$

where M^T denotes the transpose of matrix M, q_0 is consumption of the numeraire good, q_i, $i = 1, 2$ is consumption of firm i's good, q_d^i is consumption of firm d's good when it purchases the input from firm i, and matrices Q, A and B are defined as follows:

$$Q = \begin{pmatrix} q_1 \\ q_2 \\ q_d^1 \\ q_d^2 \end{pmatrix}, \quad A = \frac{1}{1-\sigma} \begin{pmatrix} 1 \\ 1 \\ \frac{\alpha(1-\sigma)+2\phi}{\beta} \\ \frac{\alpha(1-\sigma)+2\phi}{\beta} \end{pmatrix},$$

$$B = \frac{-1}{2(1-\sigma^2)} \begin{pmatrix} M_1 & M_2 \\ M_2 & ((1-\sigma^2) + 2(1 + x^2(1-\sigma) + \sigma)\frac{\phi}{\beta}) M_1 \end{pmatrix},$$

where

$$M_1 = \begin{pmatrix} 1 & \sigma \\ \sigma & 1 \end{pmatrix}, \quad M_2 = \begin{pmatrix} (1 + x(1-\sigma) + \sigma)\frac{\phi}{\beta} & (1 - x(1-\sigma) + \sigma)\frac{\phi}{\beta} \\ (1 - x(1-\sigma) + \sigma)\frac{\phi}{\beta} & (1 + x(1-\sigma) + \sigma)\frac{\phi}{\beta} \end{pmatrix}.$$

U is quasilinear. Let us show that the subutility is strictly concave. The characteristic polynomial of B can be written as:

$$P(X) = f_0 + f_1 X + f_2 X^2 + f_3 X^3 + X^4,$$

where

$$\begin{aligned} f_0 &= \frac{\beta^2(1-\sigma^2) + 4(1-x^2)\phi^2(\beta\sigma - (1-x^2)\phi^2)}{16\beta^4(1-\sigma^2)}, \\ f_1 &= \frac{\beta(1+\beta-\sigma^2) + 2(1+x^2(1-\sigma)+\sigma)\phi^2}{4\beta^3(1-\sigma^2)}, \\ f_2 &= \frac{\beta^2(\beta(4+\beta) + (1-\sigma^2)^2) + 4\beta((1-\sigma^2)(1+\sigma) + \beta + x^2(\beta + (1-\sigma)^2(1+\sigma)))\phi^2 + 4(1+x^2(1-\sigma)+\sigma)^2\phi^4}{4\beta^4(1-\sigma^2)}, \\ f_3 &= \frac{\beta(1+\beta-\sigma^2) + 2(1+x^2(1-\sigma)+\sigma)\phi^2}{\beta^2(1-\sigma^2)}. \end{aligned}$$

Given our restrictions on parameters β, σ, x and ϕ, all the coefficients of the characteristic polynomial are strictly positive. Therefore, $P(X) > 0$ for all $X \geq 0$, and matrix B is negative definite. This implies that the subutility of our representative consumer is strictly concave. It is then straightforward to show that the demand functions q_i^D, $i = 1, 2$ can be obtained by solving the program $\max_Q U(Q^T)$, subject to $q_d^1 = q_d^2 = 0$, and to the usual budget constraint and positivity constraints. Similarly, for $i \in \{1, 2\}$, demand function $q_k^{(i)}$, $k = 1, 2, d$, can be obtained by solving the representative consumer's program, subject to $q_d^j = 0$ ($j \neq i$ in $\{1, 2\}$), and to the budget and positivity constraints. □

A.15 Proof of Lemma 6

Assume, without loss of generality that firm $U1$ has merged firm firm $D1$. First, if $U1 - D1$ supplies the market at $w > m$, then, since downstream firms can switch suppliers after downstream prices are set, $U2$ can corner the upstream market and make positive profits by offering $w - \varepsilon$.

Conversely, assume that $U2$ supplies the market at $w > m$, and let us show that $U1 - D1$ wants to undercut. It is straightforward to adapt the proof of Lemma 1, to show that, if firm $U1 - D1$ sets $w - \varepsilon$, then both downstream firms accept this new offer, downstream competition is relaxed, and firm $U1 - D1$ makes positive profits: undercutting is indeed profitable.

Now, let us check that there is no equilibrium in which firms $U1 - D1$ and $U2$ share the input demand at price $w > m$. If firm Dk, $k = 2, 3$, switches from supplier $U2$ to supplier $U1 - D1$, then, firm $U1 - D1$'s first order condition shifts upward. By Lemma 7, all downstream prices go up when Dk chooses $U1 - D1$ instead of $U2$. By revealed preference, this implies that firm Dk makes larger profits when it purchases from $U1 - D1$: for the two downstream firms, purchasing from firm $U1 - D1$ is a dominant strategy. Therefore, there is no equilibrium in which firms $U1 - D1$ and $U2$ share the upstream demand at a price above marginal cost.

It is then straightforward to extend the proof of Lemma 2, to prove that the Bertrand outcome is an equilibrium. In a nutshell, starting $w_1 = w_2 = m$, an upward deviation would not affect the upstream firms' profits. A downward deviation by firm $U2$ would obviously not be profitable. And a downward deviation by firm $U1 - D1$ would be even less profitable, as the integrated firm would then make upstream losses, and make downstream competition more intense.

It remains to prove that the upstream market cannot be supplied at a price below the marginal cost. It is obvious that $U2$ never sells the input at $w < m$, otherwise it would be better off exiting the market. Assume now that $U1 - D1$ is the upstream supplier at $w < m$, and denote $U2$'s upstream offer by $w' \geq w$.[53] $U1 - D1$ is then better off exiting the upstream market, since this would shift its best response upwards, and therefore, increase all the downstream prices by Lemma 7.

A.16 Proof of Proposition 12

Proof. Assume that $\pi_i^{(i)}(w_m) \leq \pi_j^{(i)}(w_m)$, and that integrated firms do not play weakly dominated strategies on the upstream market or do not play equilibria that are Pareto-dominated by another equilibrium, and let us show that there are no equilibria with zero or one merger.

Assume by contradiction that there exists a one-merger equilibrium. Without loss of generality, assume that $U1$ merges with $D1$ in stage 1. If firms $D2$ and $D3$ do not submit positive bids in stage 2, then firm $U2$ remains independent, and, by Lemma 6, the input is priced at marginal cost. In this case, both $D2$ and $D3$ earn the same profit: Π^*, while firm $U2$ makes zero profit. But if $D2$ bids $\varepsilon > 0$ small enough, then, $U2$

[53] Actually $w' < w$. Indeed, if $w' = w$, unintegrated downstream firms strictly prefer purchasing from $D2$ since it shifts $U1 - D1$'s best response function upwards.

accepts this bid. In this case, firm $D2$'s net profit is $\Pi_{US}(w_m) - \varepsilon$ or $\Pi_{IR}(w_m) - \varepsilon$. These profits are strictly larger than Π^* for ε small enough, and therefore, this deviation is profitable for firm $D2$.

Similarly, assume that an equilibrium with no merger is profitable. Consider the following deviation: firm $D1$ bids ε in stage 1. If $U1$ refuses, it earns zero profit, by Lemma 6. Therefore, $U1$ accepts, and $D1$ and $U1$ merge. In stage 2, firms $D2$ and $D3$ have to react optimally to this deviation. Following the same argument as in the previous paragraph, a situation in which $D2$ and $D3$ bid 0 cannot be an equilibrium of the continuation subgame. Therefore, a counter-merger takes place, and firm $D1$ earns $\Pi_{US}(w_m) - \varepsilon$ or $\Pi_{IR}(w_m) - \varepsilon$: the deviation is profitable.

It is then straightforward to construct a subgame-perfect equilibrium with two mergers. Since we assume that integrated firms do not play weakly dominated strategies on the upstream market, or do not play Pareto-dominated equilibria, this implies that, in any such equilibrium, the input is sold at the monopoly upstream price. □

A.17 Proof of Proposition 14

Proof. Notice first that, with our normalization of downstream prices, consumers' surplus and welfare can be written as a function of renormalized prices as follows:

$$\begin{aligned} CS &= (1-m+\delta-c)^2 \left\{ \sum_k q_k(\tilde{p})(1-\tilde{p}_k) - \frac{1}{2}\left(\sum_k q_k(\tilde{p})\right)^2 - \frac{3}{2(1+\gamma)}\left(\sum_k q_k^2(\tilde{p}) - \frac{(\sum_k q_k(\tilde{p}))^2}{3}\right)\right\}, \\ W &= (1-m+\delta-c)^2 \left\{ \sum_k q_k(\tilde{p}) - \frac{1}{2}\left(\sum_k q_k(\tilde{p})\right)^2 - \frac{3}{2(1+\gamma)}\left(\sum_k q_k^2(\tilde{p}) - \frac{(\sum_k q_k(\tilde{p}))^2}{3}\right)\right\}. \end{aligned}$$

Consider first the one-merger case, and let us relabel firms, so that firm 1 is integrated, while firms 2 and d are non-integrated. Using again our normalization, we can write the stage 4 payoff functions as follows:

$$\begin{aligned} \tilde{\pi}_1 &= (1-m+\delta-c)^2 \left\{\tilde{p}_1 q_1(\tilde{p}) + \tilde{w}(q_2(\tilde{p}) + q_d(\tilde{p}))\right\}, \\ \tilde{\pi}_2 &= (1-m+\delta-c)^2 \left\{(\tilde{p}_2 - \tilde{w}) q_2(\tilde{p})\right\}, \\ \tilde{\pi}_d &= (1-m+\delta-c)^2 \left\{(\tilde{p}_d - \tilde{w}) q_d(\tilde{p})\right\}. \end{aligned}$$

Solving for the equilibrium downstream prices, we get $\tilde{p}_1(w) = w + (3-4w)/(6+2\gamma) - (2w)/(6+5\gamma)$, and $\tilde{p}_2(w) = \tilde{p}_d(w) = w + (3-4w)/(6+2\gamma) + w/(6+5\gamma)$. Plugging this into firm 1's payoff function yields its profit at the downstream equilibrium as a function of $\tilde{w}$:

$$\pi_1(\tilde{w}) = \frac{(3+2\gamma)(6+5\gamma)^2 + 4\tilde{w}(1+\gamma)(6+5\gamma)(18+\gamma(18+5\gamma)) - 4\tilde{w}^2(1+\gamma)(108+\gamma(198+\gamma(123+25\gamma)))}{4(3+\gamma)^2(6+5\gamma)^2}.$$

Maximizing this function in $\tilde{w}$, we get $\tilde{w}_m^1 = 1/2 - (3\gamma^2)/(216 + 2\gamma(198 + \gamma(123 + 25\gamma)))$. If $w_m^1 > m$, then, the market power of integrated firm 1 on the upstream market is constrained by the unintegrated upstream firm, and therefore, the upstream price that prevails in equilibrium is $w = m$, i.e., $\tilde{w} = \delta/(1-m+\delta-c)$. This occurs when $\tilde{w}_m^1 \geq \delta/(1-m+\delta-c)$, i.e., when δ is not too large. In this case, consumers' surplus and social welfare are given by:

$$\begin{aligned} CS_m^1 &= (1-m+\delta-c)^2 \frac{-4\tilde{w}(1+\gamma)(3+2\gamma)(6+5\gamma)^2 + (3+2\gamma)^2(6+5\gamma)^2 + 4\tilde{w}^2(1+\gamma)(54+\gamma(108+\gamma(87+25\gamma)))}{8(3+\gamma)^2(6+5\gamma)^2}, \\ W_m^1 &= (1-m+\delta-c)^2 \frac{-12\tilde{w}(1+\gamma)(6+5\gamma)^2 + (3+2\gamma)(9+2\gamma)(6+5\gamma)^2 - 4\tilde{w}^2(1+\gamma)(54+\gamma(108+\gamma(87+25\gamma)))}{8(3+\gamma)^2(6+5\gamma)^2}, \end{aligned}$$

with $\tilde{w} = \frac{\delta}{1-m+\delta-c}$. If instead $\tilde{w}_m^1 \leq \delta/(1-m+\delta-c)$, firm 1 sets its monopoly upstream price on the input market, and

$$
\begin{aligned}
CS_m^1 &= (1-m+\delta-c)^2 \frac{5832+\gamma(22680+\gamma(37584+\gamma(34056+\gamma(17811+25\gamma(204+25\gamma)))))}{8(108+\gamma(198+\gamma(123+25\gamma)))^2}, \\
W_m^1 &= (1-m+\delta-c)^2 \frac{25272+\gamma(96552+\gamma(155520+\gamma(134784+\gamma(66081+25\gamma(692+75\gamma)))))}{8(108+\gamma(198+\gamma(123+25\gamma)))^2}.
\end{aligned}
$$

It is then straightforward to show that, when firm 1's offer is constrained, the derivatives of consumers' surplus and welfare with respect to $\tilde{w}$ are negative for $\tilde{w} = 0$ (i.e., $w = m - \delta$) $\tilde{w} = \tilde{w}_m^1$. Since these functions are quadratic in $\tilde{w}$, this means that consumers' surplus and welfare decrease in $\tilde{w}$ for relevant values of $\tilde{w}$.

In the two-merger subgame, we can take the equilibrium downstream prices and the monopoly upstream price from the proof of Proposition 3, and plug them in the expressions of consumers' surplus and welfare. When $\gamma < \overline{\gamma}$, the input is priced at marginal cost, and consumers' surplus and welfare are equal to $(1-m+\delta-c)^2(3+2\gamma)^2/(8(3+\gamma)^2)$ and $(1-m+\delta-c)^2(1/2-9/(8(3+\gamma)^2))$ respectively. The difference between consumers' surplus (resp. welfare) in the two-merger subgame and consumers' surplus (resp. welfare) in the one-merger subgame is therefore equal to $(1-m+\delta-c)^2$ times a function which is increasing in $\tilde{w}$. When $\tilde{w} = 0$, this difference is obviously zero (in both subgames, the input is supplied at the same marginal cost). Therefore, this difference is strictly positive for all $\tilde{w} > 0$, and hence, for all $\delta > 0$.

Assume now that $\gamma \geq \overline{\gamma}$. In this case, there is a monopoly-like equilibrium on the upstream market, and we get:

$$
\begin{aligned}
CS_m^2 &= (1-m+\delta-c)^2 \frac{314928+1434672\gamma+2827548\gamma^2+3133404\gamma^3+2120229\gamma^4+888102\gamma^5+221769\gamma^6+29640\gamma^7+1600\gamma^8}{8(648+\gamma(1296+\gamma(909+\gamma(249+20\gamma))))^2}, \\
W_m^2 &= (1-m+\delta-c)^2 \frac{1084752+4583952\gamma+8272692\gamma^2+8257788\gamma^3+4930551\gamma^4+1775322\gamma^5+368499\gamma^6+39240\gamma^7+1600\gamma^8}{8(648+\gamma(1296+\gamma(909+\gamma(249+20\gamma))))^2}.
\end{aligned}
$$

Again, the difference between consumers' surplus (resp. welfare) in the two-merger subgame and consumers' surplus (resp. welfare) in the one-merger subgame is equal to $(1-m+\delta-c)^2$ times a function which is increasing in $\tilde{w}$. This difference is obviously negative when $\tilde{w} = 0$, and straightforward computations show that it is positive for $\tilde{w} = \tilde{w}_m^1$. Therefore, there exists a threshold such that consumers' surplus (resp. welfare) are larger in the two-merger subgame iff $\tilde{w}$ is above this threshold. Since $\tilde{w}$ increases in δ, there is also a threshold such that the second merger increases consumers' surplus (resp. welfare) iff δ is above this threshold.

These thresholds can then be computed, and we show numerically that they are strictly decreasing in γ, and that the welfare threshold is always above the consumers' surplus threshold. A mathematica file with all these computations is available online. □

A.18 Proof of Proposition 15

Proof. As proven elsewhere, we can renormalize all costs to 0 without loss of generality. Consider first the no-merger subgame, and assume that firm 1 is the upstream supplier at price w. We can take the expressions of profits and prices at the downstream equilibrium from the proof of Proposition 3. The joint profit of firms 1 and d is given by:

$$
\frac{9w\gamma(1+\gamma)^2(6+5\gamma)+3(3+2\gamma)(6+5\gamma)^2-w^2(1+\gamma)(162+\gamma(324+\gamma(252+\gamma(81+7\gamma))))}{6(3+\gamma)^2(6+5\gamma)^2}.
$$

The above function is strictly concave in w, and reaches its maximum when the input price is equal to

$$w_{tp} \equiv \frac{9\gamma(1+\gamma)(6+5\gamma)}{2(162+\gamma(324+\gamma(252+\gamma\ (81+7\gamma))))}.$$

Plugging this value of w into the equilibrium downstream prices given in Section A.7, we can then compute firms 1 and d's joint profit, social welfare, and consumers' surplus:

$$\begin{aligned}
JP_{tp} &= \frac{216+432\gamma+315\gamma^2+83\gamma^3}{8(162+324\gamma+252\gamma^2+81\gamma^3+7\gamma^4)}, \\
W_{tp} &= \frac{314928+1318032\gamma+2434860\gamma^2+2561868\gamma^3+1654803\gamma^4+658314\gamma^5+152559\gamma^6+17928\gamma^7+784\gamma^8}{32(162+324\gamma+252\gamma^2+81\gamma^3+7\gamma^4)^2}, \\
CS_{tp} &= \frac{104976+478224\gamma+959364\gamma^2+1099332\gamma^3+780381\gamma^4+347454\gamma^5+93249\gamma^6+13488\gamma^7+784\gamma^8}{32(162+324\gamma+252\gamma^2+81\gamma^3+7\gamma^4)^2},
\end{aligned}$$

where JP stands for 'joint profit', and subscript tp stands for 'two-part'.

If firms 1 and d merge, downstream payoff functions are given by: $\tilde{\pi}_{1d} = p_1q_1 + p_dq_d$ and $\tilde{\pi}_2 = p_2q_2$. The downstream equilibrium prices are equal to $p_1 = p_d = (6+5\gamma)/(2(6+6\gamma\gamma^2))$ and $p_2 = (3+2\gamma)/(2(6+6\gamma\gamma^2))$. Therefore,

$$\begin{aligned}
JP_H &= \frac{(3+\gamma)(6+5\gamma)^2}{18(6+6\gamma\gamma^2)^2}, \\
W_H &= \frac{486+1044\gamma+765\gamma^2+215\gamma^3+18\gamma^4}{36(6+6\gamma\gamma^2)^2}, \\
CS_H &= \frac{(3+\gamma)(3+2\gamma)(18+\gamma(26+9\gamma))}{36(6+6\gamma\gamma^2)^2},
\end{aligned}$$

where subscript H stands for'horizontal merger'. Comparing these values with the ones derived in the no-merger subgame, we get:

$$\begin{aligned}
JP_H - JP_{tp} &= \frac{972\gamma^2+2916\gamma^3+3024\gamma^4+1224\gamma^5+81\gamma^6-47\gamma^7}{72(6+6\gamma+\gamma^2)^2(162+324\gamma+252\gamma^2+81\gamma^3+7\gamma^4)}, \\
W_H - W_{tp} &= \big(-3779136\gamma-16061328\gamma^2-28028592\gamma^3-23978268\gamma^4-6998400\gamma^5+5219640\gamma^6+6142392\gamma^7+2687013\gamma^8 \\
&\quad +580122\gamma^9+55233\gamma^{10}+1552\gamma^{11}\big)\ /\ \big(288(6+6\gamma+\gamma^2)^2(162+324\gamma+252\gamma^2+81\gamma^3+7\gamma^4)^2\big), \\
CS_H - CS_{tp} &= \big(-3779136\gamma-17950896\gamma^2-36216720\gamma^3-39007332\gamma^4-21741696\gamma^5-2455272\gamma^6+4854168\gamma^7+3452139\gamma^8 \\
&\quad +1073214\gamma^9+162351\gamma^10+9368\gamma^11\big)\ /\ \big(288(6+6\gamma+\gamma^2)^2(162+324\gamma+252\gamma^2+81\gamma^3+7\gamma^4)^2\big).
\end{aligned}$$

The first expression is negative for γ high enough. It has only one positive root: $\gamma_H \simeq 6.97309$. Therefore, a horizontal merger takes place if and only if $\gamma \geq \gamma_H$. The second (resp. third) expression is positive for γ high enough. It has only one positive root: $\gamma_W \simeq 1.57682$ (resp. $\gamma_{CS} \simeq 1.90188$). Therefore, a horizontal merger increases social welfare (resp. consumers' surplus) if and only if $\gamma \geq \gamma_W$ (resp. $\gamma \geq \gamma_{CS}$).

□

References

Avenel, E., and C. Barlet (2000): "Vertical Foreclosure, Technological Choice and Entry on the Intermediate Markets," *Journal of Economics and Management Strategy*, 9(2), 211–230.

Bonanno, G., and J. Vickers (1988): "Vertical Separation," *Journal of Industrial Economics*, 36(3), 257–265.

Bork, R. H. (1978): *The Antitrust Paradox: A Policy at War with Itself.* New York: Basic Books.

Chen, Y. (2001): "On Vertical Mergers and Their Competitive Effects," *RAND Journal of Economics*, 32(4), 667–85.

Chen, Y., and M. H. Riordan (2007): "Vertical Integration, Exclusive Dealing, and Ex Post Cartelization," .

Choi, J. P., and S.-S. Yi (2000): "Vertical Foreclosure with the Choice of Input Specifications," *RAND Journal of Economics*, 31(4), 717–743.

Deneckere, R., and C. Davidson (1985): "Incentives to Form Coalitions with Bertrand Competition," *RAND Journal of Economics*, 16(4), 473–486.

EC (2007): "Guidelines on the assessment of non-horizontal mergers under the Council Regulation on the control of concentrations between undertakings," Discussion paper, European Commission.

European Commission (2008): *Competition Policy Newsletter*chap. Digital Maps Go Vertical: TomTom/Tele Atlas and Nokia/NAVTEQ. 2008-3.

Fauli-Oller, R., and J. Sandonis (2002): "Welfare Reducing Licensing," *Games and Economic Behavior*, 41(2), 192–205.

FTC (1997): "Horizontal Merger Guidelines," Discussion paper, U.S. Department of Justice and Federal Trade Commission.

Fudenberg, D., and J. Tirole (1984): "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look," *American Economic Review*, 74(2), 361–66.

Hart, O., and J. Tirole (1990): "Vertical integration and market foreclosure," *Brookings Papers on Economic Activity*, 1990, 205–276.

Höffler, F. (2008): "On the Consistent Use of Linear Demand Systems if not All Varieties Are Available," *Economics Bulletin*, 4(14), 1–5.

Höffler, F., and K. Schmidt (2008): "Two Tales on Resale," *International Journal of Industrial Organization*, 26, 1448–1460.

Kreps, D. M., and J. A. Scheinkman (1983): "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics*, 14(2), 326–337.

Marx, L. M., and G. Shaffer (2004): "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity: Comment," *American Economic Review*, 94, 796–801.

McAfee, R. P., and M. Schwartz (1994): "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity," *American Economic Review*, 84(1), 210–30.

NOCKE, V., AND L. WHITE (2007): "Do Vertical Mergers Facilitate Upstream Collusion?," *American Economic Review*, 97(4), 1321–1339.

NORMANN, H.-T. (2009): "Vertical Integration, Raising Rivals' Costs and Upstream Collusion," *European Economic Review*, forthcoming.

O'BRIEN, D. P., AND G. SHAFFER (1992): "Vertical Control with Bilateral Contracts," *RAND Journal of Economics*, 23(3), 299–308.

ORDOVER, J., AND G. SHAFFER (2007): "Wholesale Access in Multi-Firm Markets: When Is It Profitable to Supply a Competitor?," *International Journal of Industrial Organization*, 25(5), 1026–1045.

ORDOVER, J. A., G. SALONER, AND S. C. SALOP (1990): "Equilibrium Vertical Foreclosure," *American Economic Review*, 80, 127–142.

POSNER, R. A. (1976): *Antitrust Law: An Economic Perspective*. Chicago: University of Chicago Press.

REIFFEN, D. (1992): "Equilibrium Vertical Foreclosure: Comment," *American Economic Review*, 82(3), 694–97.

REY, P., AND J. TIROLE (2005): *Handbook of Industrial Organization, vol. III* chap. A Primer on Foreclosure, pp. 456–789. North Holland.

REY, P., AND T. VERGÉ (2004): "Bilateral Control with Vertical Contracts," *RAND Journal of Economics*, 35(4), 728–746.

RIORDAN, M. H. (2008): *Handbook of Antitrust Economics, Paolo Buccirossi* chap. Competitive Effects of Vertical Integration. MIT Press.

SALINGER, M. A. (1988): "Vertical Mergers and Market Foreclosure," *Quarterly Journal of Economics*, 103, 345–356.

SALOP, S. C. (1979): "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10(1), 141–156.

SHUBIK, M., AND R. LEVITAN (1980): *Market Structure and Behavior*. Cambridge: Harvard Press.

VIVES, X. (1999): *Oligopoly Pricing: Old Ideas and New Tools*. The MIT Press, Cambridge, Massachusetts.